

# Essays on Debt Maturity

by

Wei Wei

Business Administration  
Duke University

Date: \_\_\_\_\_

Approved:

---

Adriano Rampini, Co-Supervisor

---

S. Viswanathan, Co-Supervisor

---

Simon Gervais

---

Daniel Xu

Dissertation submitted in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy in Business Administration  
in the Graduate School of Duke University  
2014

# ABSTRACT

## Essays on Debt Maturity

by

Wei Wei

Business Administration  
Duke University

Date: \_\_\_\_\_

Approved:

---

Adriano Rampini, Co-Supervisor

---

S. Viswanathan, Co-Supervisor

---

Simon Gervais

---

Daniel Xu

An abstract of a dissertation submitted in partial fulfillment of the requirements for  
the degree of Doctor of Philosophy in Business Administration  
in the Graduate School of Duke University  
2014



# Abstract

I study firms' debt maturity decisions. I provide two models for optimal debt maturity choices when facing stochastic productivity and rollover risk. The first model is based on firms' need to smooth their capital when facing uncertainties in external financing. When the capital market freezes, new external financing is difficult. Firms with large debt repayments due have to forego good investment opportunities and in severe cases cut back on dividends. Long-term debt reduces immediate repayments and allows firms to keep the borrowed capital for future production. Therefore, when freezes are likely, firms respond by using more long-term financing and are better prepared. However, when the probability of freezes is low, firms turn to short-term financing. When a freeze suddenly occurs, the impact is significant and costly. The model predicts that constrained firms use more short-term debt. Based on the model, I propose investment-debt sensitivity as a new measure for financial constraints.

The second model depicts an economy in which entrepreneurs reallocate capital resources through borrowing and lending in either short-term or long-term debt. In expansions, productivity is more persistent and uncertainty in productivity is low, so entrepreneurs can better predict their future prospects. Hence, they choose to use more long-term debt to finance their productions. In recessions, future prospects are less clear to the entrepreneurs; therefore, they choose to use more short-term debt. The model explains the documented facts on pro-cyclical debt maturity in the economy. It also highlights that the shortening debt maturity structure causes capital resources to be less efficiently allocated in recessions further exacerbates the

bad times. I argue that the change in the predictability of TFP drives pro-cyclical debt maturity, and that the maturity structure further amplifies the fluctuations in aggregate production.

To my parents, Bing Wei and Lanzhen Song.

# Contents

<b>Abstract</b>	<b>iv</b>
<b>List of Tables</b>	<b>ix</b>
<b>List of Figures</b>	<b>x</b>
<b>Acknowledgements</b>	<b>xi</b>
<b>1 Rollover Risk and Debt Maturity</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Environment . . . . .	5
1.2.1 Debt financing . . . . .	6
1.2.2 Firm's problem . . . . .	7
1.3 Capital and Dividend Policy: . . . . .	13
1.4 Short-term v.s. Long-term debt . . . . .	16
1.4.1 Cash as negative short-term debt . . . . .	16
1.4.2 Optimal long-term debt to capital ratio . . . . .	17
1.5 Fixed Cost of Long-term Debt Issuance . . . . .	19
1.6 Current v.s. Future Investments and Debt Maturity . . . . .	21
1.7 Empirical Testing . . . . .	27
1.7.1 Financial constraints and debt maturity . . . . .	27
1.7.2 Investment-debt sensitivity . . . . .	29
1.7.3 Investment-debt sensitivity and the natural experiment . . . . .	32
1.8 Related Literature . . . . .	32

1.9	Conclusion . . . . .	36
<b>2</b>	<b>Resource Allocation and Debt Maturity</b>	<b>53</b>
2.1	Introduction . . . . .	53
2.2	Environment . . . . .	57
2.3	Individual Problem . . . . .	59
2.3.1	Capital, labor, and total debt . . . . .	61
2.3.2	Consumption, net worth and long-term debt . . . . .	62
2.4	Equilibrium and Dynamics . . . . .	66
2.5	Debt Maturity and TFP losses . . . . .	71
2.5.1	Importance of long-term debt . . . . .	71
2.5.2	Pro-cyclical Debt Maturity . . . . .	73
2.6	Conclusion . . . . .	74
<b>A</b>	<b>Proofs</b>	<b>84</b>
	<b>Bibliography</b>	<b>109</b>
	<b>Biography</b>	<b>113</b>



# List of Tables

1.1	Summary Statistics. . . . .	45
1.2	Financial Constraints and Debt Maturity. . . . .	46
1.3	Investment Regression Full Panel. . . . .	47
1.4	Investment Regression Whited Wu Groups. . . . .	48
1.5	Investment Regression Size Groups. . . . .	49
1.6	Investment Regression Dividend Groups. . . . .	50
1.7	Investment Regression Reduction in Tariff. . . . .	51
1.8	Definition of Variables. . . . .	52
2.1	Persistence of Idiosyncratic Productivity. . . . .	81
2.2	Effects of Long-term Debt. . . . .	82
2.3	Growth for In and Out of the Market. . . . .	83

# List of Figures

1.1	Capital Choices. . . . .	38
1.2	Dividends Payouts. . . . .	39
1.3	Dividends Payout Region. . . . .	40
1.4	Long-term v.s. Short-term Debt. . . . .	41
1.5	Debt Policies with Fixed Costs. . . . .	42
1.6	Contour of Debt Policies with Fixed Costs. . . . .	43
1.7	Value Functions for Two Long-term Debt Levels. . . . .	44
2.1	The Entrepreneur's Problem (Timeline). . . . .	76
2.2	Spectrum of Entrepreneurs in the Market. . . . .	77
2.3	Evolution of the Economy. . . . .	78
2.4	Wealth Shares and Long-term Debt Policy. . . . .	79
2.5	Wealth Shares In and Out of Market. . . . .	80

# Acknowledgements

I am greatly indebted to Simon Gervais, Adriano Rampini, S. “Vish” Viswanathan, and Daniel Xu for their encouragement, support, and guidance. I thank John Graham, Lukas Schmid, Barney Hartman-Glaser, and Ming Yang for helpful discussions and comments. I am also grateful to Shiming Fu, Hyunseob Kim, Alex Young, Felix Feng, Yan Liu, and Basil Williams for helpful discussions and conversations.

# Rollover Risk and Debt Maturity

## 1.1 Introduction

In the aftermath of the last recession, the government, the media, and the academia all blamed short-term financing for causing and exacerbating the crisis. However, it is important to understand why firms choose to take on so much short-term debt in the first place. In this chapter, I provide a dynamic model of firm financing with collateralized short and long-term debt in an environment with potential market freezes. Firms may choose to borrow short-term debt which requires a lower interest rate than long-term debt. However, they will be more exposed to rollover risk when the debt repayment is due and the market for external financing freezes. Hence, firms' debt maturity decisions depend on the likelihood of a market freeze. The model points out that when the probability of a financial market freeze is viewed to be low, firms optimally switch to short-term financing. As a consequence they become more vulnerable to adverse shocks to external financing. Consequently, when the capital market suddenly freezes, the effects are devastating.

Prior literature on capital structure linked firms' new investment and financing decisions to their existing financial characteristics. However, when a firm tries to raise

capital through external financing, the (shadow) cost of borrowing usually depends on both the firm's characteristics and financial market conditions. For example, when the firm has a weak balance sheet or dim future profitability, its cost of borrowing will be high. Equally important, if the overall financing condition is severe, like when the capital market collapses, the cost of borrowing will be high. Therefore, the firm has to manage the accumulation of net worth while taking into account the possible disruptions of financing shocks. In recent years, many papers have focused on explaining how financial crises occur. In particular some focus on how short-term debt structure leads to market freezes (see, for example, Acharya, Gales, and Yorulmazer (2011)). In this chapter, I do not try to model how market freezes come about, but rather treat it as an exogenous shock to the firm. This allows me to focus on the firm's response to the risk of not being able to borrow when a freeze occurs.

The main contribution of my model is to recognize that debt maturity structures play an important role in balancing firm growth and hedging for rollover risk caused by an adverse shock to external capital markets. Essentially, a long debt maturity structure reduces the repayments due for each period. In the extreme case, with perpetual long-term debt, firms never have to repay the principal of the debt, thus they can keep the capital in production for all periods. On the other hand, short-term debt demands a low interest rate but needs to be refinanced more often. Hence, short-term debt facilitates the accumulation of net worth in the current period while exposing the firm to higher future rollover risk when future financing is costly. This trade off has important implications.

The model predicts that more constrained firms choose to use more short-term debt in order to facilitate current investment and grow net worth faster. Using a quasi-natural experiment from decreases in tariffs, I confirm that when firms become more constrained due to exogenous increases to competition, they use more short-term debt. On average, firms that experience abnormal increases in competition reduce their

fraction of long-term (maturing in more than 3 year) debt by 7.9%. Or equivalently, the firms reduce their average debt maturity from 7.5 years to 7 years after the increase in competition.<sup>1</sup> Moreover, I propose a new measure for financial constraints based on the fact that debt repayment should have a large impact on investment when firms have difficulties raising new capital. In essence, more constrained firms have a bigger sensitivity of investment to debt repayment. Hence, this investment-debt sensitivity can be a good proxy for financial constraints. In the data, one standard deviation increase in debt repayment leads to a more than 10% reduction in the investment rate for the constrained firms.<sup>2</sup> The same increase in debt repayment only leads to a less than 3% reduction in the investment rate for the relatively unconstrained firms. Moreover, the investment-debt sensitivity measure is unlikely to suffer from the endogeneity problem that undermines the validity of investment-cash flow sensitivity.<sup>3</sup> Finally, the investment-debt sensitivity performs well for all subperiods from 1967 to 2011. This means that the measure is reliable through time.

Unlike any of the existing models which use a constant maturity structure decided at time 0, my model allows the firm to adjust its level of short and long-term debt every period according to its new financial strength and new capital market conditions. Despite its dynamic nature, the model is simple and tractable. Hence, it allows for rich characterizations of the firm's dividends, investments, short-term and long-term financing decisions. The model highlights the effects of firms' current net worth and existing long-term debt on their dividend payout and new debt issuance policies. It predicts that firms will use more long-term debt when facing better future

---

<sup>1</sup>If we assume that all debts maturing within 3 years have an average maturity of 2 years and all debts maturing after 3 years have an average maturity of 15 years, then the average debt maturity is 7.5 years. Under the same assumption, firms shorten their debt maturity by half a year after increases in competition.

<sup>2</sup>The investment rate is the ratio of new investment to existing capital. It is calculated as new capital expenditure divided by current net property, plant and equipment.

<sup>3</sup>When cash flows are persistent and when marginal Q fails to capture all information on investment opportunities, current cash flows are naturally correlated with investments. Also, see Erickson and Whited (2000, 2012) for measurement errors in marginal Q.

productivity. It also predicts that holding net worth constant, firms with proportionally more long-term debt pay more dividends. I try to test those implications with a quasi-natural experiment from exogenous drops in import tariffs. With fixed costs, the model explains why firms sometimes choose to save cash instead of paying down outstanding debt. A two period version of the model also allows me to explore how the changes in future investment perspectives affect the current trade-off between short and long-term debt.

The collateral constraints in my model are closely related to those derived from limited contract enforcement. Unlike in Kehoe and Levine (1993, 2001, 2008), the firm in my model can run away with a fraction of the capital without being excluded from external capital markets. Rampini and Vishwanathan (2010, 2013) give detailed derivations of collateral constraints from a dynamic model with limited enforcement. With complete markets, the firm in their model can engage in risk managements by contracting against a particular future state. However, the firm in my model can only borrow non-state contingent debt. In this aspect, my collateral constraints are similar to those in Kiyotaki and Moore (1997), except there are two types of debt and they both have to be collateralized against the firms' capital. This collateral constraint fixes the maximum leverage that can be achieved by any mix of short and long-term borrowing. It provides a clean and tractable way of modeling the advantage of short-term debt over long-term debt in raising capital because short-term debt permits higher leverage. My model offers a starting point for models that allow for dynamic maturity choices.

The chapter is organized as follows. In Section 1.2, I first introduce the baseline model and prove some standard results and properties. I also derive the optimality conditions and give interpretations. In Section 1.3, I characterize the firm's dividend policy. In particular I show that under some financing conditions, firms' dividend policies depend on both their net worth and existing debt. In Section 1.4, I discuss

about the effect of the financing shock and describe the optimal long-term debt choices for firms with different levels of net worth. In Section 1.5, I show some numerical results and give some intuitions about firms' optimal policy when there is fixed cost for issuing long-term debt and how it differs from the baseline model. In Section 1.6, I present a 2-period model and explain the effects of investment opportunity on firms' debt maturity choices. In Section 1.7, I provide some empirical results that verify my model predictions and propose a new measure for financial constraints. In Section 1.8, I relate my work to existing literature. Finally, in Section 1.9, I summarize the main results. The proofs are presented in the appendix.

## 1.2 Environment

I consider an environment with two independent exogenous shocks that affect a firm's debt capacity and investment needs. First, with probability  $\pi$ , the capital market freezes, hence the firm is unable to raise new capital. The assumption that the firm loses access to external market with probability  $\pi$ , is merely a simplified way of modeling financing shocks. In practice, the cost of external financing may fluctuate exogenously (to the firm) when the lenders experience some supplier side shocks. Hence, the transaction fee, the price, the interest rate, or the terms (such as maturity or covenants) on a debt may vary from time to time even if the firm's characteristics stay the same. For simplicity, I lump all those adverse changes into the inability of borrowing new debt. Of course, it is more or less the most extreme case of financing shock.

Second, the firm's productivity is stochastic. In each period, the productivity realizes after the firm makes investment and financing decisions. When experiencing low productivity realizations, the firm is short on internal resources because cash flows are low. Consequently, it has to borrow more to maintain the same level of production. When external financing is prevented by market freezes, the firm is forced



to downsize and forgo good investment opportunities.

### *1.2.1 Debt financing*

In this partial equilibrium model, the lenders have deep pockets in all times and states. They are willing to lend in long-term at a return  $R_L$  and in short-term at a return  $R_S < R_L$ . Importantly, I assume that debt contracts cannot be written contingent on the realizations of the two financing shocks.

Moreover, I assume that the firm can not save by lending in long term at rate  $R_L$  to prevent any firm from borrowing at  $R_S$  and lending at  $R_L$  simultaneously. Equivalently, I can assume that the collateralizability of the long-term lending is very low due to illiquidity in the secondary market for long-term debt. Hence, saving through long-term lending significantly reduces the amount that the firm can borrow with short-term debt. Since the firm's production technology is average sufficiently good, it will not try to save through long-term lending.

On the other hand, the firm can borrow long-term debt at a higher rate  $R_L$ , and save the unused resources at a low rate  $R_S$ . Since long-term debt is valuable in terms of helping the firm in endure times with refinancing difficulties, the firm might want to keep the same level of long-term debt even when experiences bad productivity shock currently. However, in order for the firm to satisfy the collateral constraint, it has to now save in short-term and use that saving as collateral. Alternatively, we can view it as that the firm needs to save cash instead of investing in capital in order not to violate some covenants from long-term debt. For example, saving cash instead of investing in fixed capital can help the firm maintaining a minimal current ratio, especially when the firm experiences low recent cash flows.

In practice, suppose the firm invests in any type T-bill or money market funds, then the amount will be deducted from short-term debt. Since those saving vehicles are highly liquidity and bear low interest rates and since the firm can choose to sell

those investment at any time to invest in capital or pay down debt, those savings will be considered negative short-term debt regardless their actual maturities.

Finally, if the firm does not have enough cash flow to pay for its debt obligation, it can always choose to sell its capital. The capital depreciates at rate  $\delta$  per period. Also, the firm can always run away with all cash flows and a fraction  $\theta$  of the remaining capital. Hence, all liabilities of the firm have to be collateralized by its capital. More specifically, I implement the following constraint:

$$\theta(1 - \delta)k \geq R_S b_S + R_L b_L, \quad (1.1)$$

The constraint requires the collateralizable fraction,  $(1 - \theta)$ , of the un-depreciated capital,  $(1 - \delta)k$ , to be greater or equal to the total amount of interests and principal repayments for both long-term and short-term debt. With this collateral constraint, the firm never defaults. I can assume the prices of both types of debt to be one instead of having endogenous bond prices derived from probabilities of defaults. This makes the model more tractable. Also, collateral constraint can be derived from a micro-problem of limited enforcement. Rampini and Viswanathan (2010) show that in their set up limited enforcement is equivalent to one-period state-contingent debt subject to collateral constraints. In practice firms do either implicitly or explicit borrow against their tangible assets (see Rampini and Viswanathan (2013) for the importance of tangible assets and collateralized financing).

### 1.2.2 Firm's problem

The firm's problem is as follows. At the beginning of each period, the firm learns whether the financing shock realizes (i.e.  $s = 0$  or  $1$ ) before making any investment and financing decision. If the financing shock occurs ( $s = 0$ ), then it will not be able to raise new capital (though either short-term or long-term debt). Given the financing conditions, productivity state, its current net worth  $w$ , and its existing long-term debt  $b_L^-$ , the firm chooses dividend  $d$ , capital  $k$ , next period net worth  $w(z')$  for each state

$z'$ , and non-state-contingent short-term and long-term debt  $b_S$  and  $b_L$ , to maximize the discounted expected value of dividends:

$$V(w, b_L^-, z, s) \equiv \max_{\{d, k, b_L, w(z'), b_S\} \in \mathbb{R}_+^{3+Z} \times \mathbb{R}} d + \beta \int \left\{ \pi V(w(z'), b_L, z', 0) + (1 - \pi) V(w(z'), b_L, z', 1) \right\} dQ_Z, \quad (1.2)$$

subject to the budget constraints

$$w + b_S + b_L \geq d + k \quad (1.3)$$

$$A(z')f(k) + (1 - \delta)k - R_S b_S - R_L b_L \geq w(z') \quad \forall z' \in Z, \quad (1.4)$$

the collateral constraint

$$\theta(1 - \delta)k \geq R_S b_S + R_L b_L, \quad (1.5)$$

and the financing constraints

$$b_L^- \geq b_L, \quad \text{for } s = 0, \quad (1.6)$$

$$0 \geq b_S, \quad \text{for } s = 0, \quad (1.7)$$

In the program in (1.2)-(1.7), due to limited liability and since I do not consider equity issuance, dividends  $d$  and net worth  $w(z')$  are non-negative. Moreover, capital  $k$  and long-term debt  $d$  are also required to be non-negative. I write the budget constraints (1.3) and (1.4) as inequality constraints to make the constrained set convex, despite the fact that both constraints are binding for any optimal policy. There are two endogenous state variables, net worth  $w$  and total long-term debt outstanding in last period  $b_L^-$  (here after, I refer to it as existing long-term debt). However, by setup, the optimal policy does not depend on existing long-term debt  $b_L^-$  when the firm has access to external market (i.e.  $s = 1$ ). Indeed, the variable  $b_L^-$  is not present in the value function nor in the constraints when the firm has access to external market

since the firm can freely adjust its capital and debt structure. Hence  $b_L^-$  is only a state variable when financing shock occurs (i.e.  $s = 0$ ).

I also require that production function and productivity shocks to satisfy the following assumptions.

**Assumption 1.**  *$f$  is strictly increasing, strictly concave,  $f(0) = 0$ ,  $\lim_{k \rightarrow 0} f_k(k) = +\infty$ , and  $\lim_{k \rightarrow +\infty} f_k(k) = 0$ .*

**Assumption 2.** *For all  $z, \hat{z} \in Z$  such that  $\hat{z} > z$ , (i)  $A(\hat{z}) > A(z)$  and (ii)  $A(z) > 0$ .*

To simplify notations, I use  $x$  to denote the choice variables,  $x \equiv [d, k, b_L, w(z'), b_S]$ , and use  $\Gamma(w, b_L^-, z, s)$  to denote the constraint set given state variables  $w, b_L^-, z$ , and  $s$ . Thus,  $\Gamma(w, b_L^-, z, s)$  is the set of  $x \in \mathfrak{R}_+^{3+Z} \times \mathfrak{R}$  such that (1.3)-(1.7) are satisfied.

Also I define an operator  $T$  as

$$(Tf)(w, b_L^-, z, s) = \max_{x \in \Gamma(w, b_L^-, z, s)} d + \beta \int \left\{ pV(w(z'), b_L, z', 0) + (1-p)V(w(z'), b_L, z', 1) \right\} dQ_Z$$

In the proposition below, I prove that the firm's problem is well behaved and has an unique fixed value function  $V$ .

**Proposition 1.** *(i)  $\Gamma(w, b_L^-, z, s)$  is convex, given  $(w, b_L^-, z, s)$ , and jointly convex in  $w, b$ . (ii)  $\Gamma(w, b_L^-, z, s)$  is monotone (increasing) in  $w$  in the sense that  $w \leq \hat{w}$  implies  $\Gamma(w, b_L^-, z, s) \subseteq \Gamma(\hat{w}, b_L^-, z, s)$ .  $\Gamma(w, b_L^-, z, s)$  is monotone (decreasing) in  $b_L^-$  in the sense that  $b_L^- \leq \hat{b}_L^-$  implies  $\Gamma(w, \hat{b}_L^-, z, s) \subseteq \Gamma(w, b_L^-, z, s)$ . (iii) The operator  $T$  satisfies Blackwell's conditions and there exists a unique fixed point  $V$ . (iv)  $V$  is continuous, strictly increasing in  $w$ .  $V$  is continuous, decreasing in  $b$ . And  $V$  is jointly concave in  $w$  and  $b$ . (v)  $V(w, b_L^-, z, s)$  is strictly concave in  $w$  for  $w \in \text{int}\{w : d(w, b_L^-, z, s) = 0\}$ . When  $s = 0$  and  $b^- < \frac{1-p_L}{p_L}w$ ,  $V(w, b_L^-, 1, 0)$  is strictly (jointly) concave in  $w$  and  $b_L^-$  for  $\{w, b_L^-\} \in \text{int}\{\{w, b_L^-\} : d(w, b_L^-, z, 0) = 0, b^- < \frac{1-p_L}{p_L}w\}$ .*

Let  $\mu$ ,  $\beta\mu(z')\pi(z, z')$ ,  $\beta\lambda$ ,  $\gamma_l$ , and  $\gamma_s$  be the multipliers on constraints (1.3), (1.4), (1.5), (1.6), and (1.7) respectively. Also, let  $\nu_d$  and  $\nu_{b_L}$  be the multipliers on the constraints  $d \geq 0$  and  $b_L \geq 0$ . Finally, I will show in Lemma 6 that  $k > 0$  and  $w(z') > 0 \forall z' \in Z$ .

From the envelop conditions and FOC with respect to  $d$ , I derive the following equations:

$$V_1(w, b_L^-, z, s) = \mu = 1 + \nu_d. \quad (1.8)$$

$$V_2(w, b_L^-, z, 0) = \gamma_l$$

Hence, the marginal value of (current) net worth is equal to the value of dividend. The marginal value of existing long-term debt is equal to the multiplier on the credit constraint (1.6) when  $s = 0$ . In another word, the value of having additional long-term debt is that it relaxes the credit constraint on long-term debt. However, when  $s = 1$ , there is no value of having long-term debt, since the firm can borrow new long-term debt anyway.

The capital  $k$ , short-term debt  $b_S$ , and long-term debt  $b_L$  decisions are governed by the following FOCs respectively:

$$\begin{aligned} \frac{\partial V}{\partial k} : \mu = \beta \int [A(z')f_k(k) + (1 - \delta)] & \left\{ \pi\mu(w(z'), b_L, z', 0) + (1 - \pi)\mu(w(z'), b_L, z', 1) \right\} dQ_Z \\ & + \beta\theta(1 - \delta)\lambda \end{aligned} \quad (1.9)$$

$$\frac{\partial V}{\partial b_S} : \mu = \beta \int \left\{ \pi\mu(w(z'), b_L, z', 0) + (1 - \pi)\mu(w(z'), b_L, z', 1) \right\} dQ_Z R_S + \beta\lambda R_S + \gamma_s 1_{s=0} \quad (1.10)$$

$$\begin{aligned} \frac{\partial V}{\partial b_L} : \mu = \beta \int & \left\{ \pi\mu(w(z'), b_L, z', 0) + (1 - \pi)\mu(w(z'), b_L, z', 1) \right\} dQ_Z R_L + \beta\lambda R_L + \gamma_l 1_{s=0} \\ & - \beta\pi \int_{z' \geq \bar{z}} \gamma_l(w(z'), b_L, z', 0) dQ_Z - \nu_{b_L} \end{aligned} \quad (1.11)$$

The first two FOCs are quite standard except that when  $s = 0$ , the firm may be constrained because it can not raise new capital. The shadow cost of that constraint is  $\gamma_s$ . The last equation states that when the firm chooses new long-term debt level, it also takes into account the value the long-term debt might have in future when the financing shock hits. More specifically, that value is  $\beta\pi \int_{z' \geq \bar{z}} \gamma_l(w(z'), b_l, z', 0) dQ_Z$  which depends on both the probability of financing shock  $\pi$  and the realizations of the productivity shock  $z'$ . Also, note that when net worth is too low, the collateral constraint will bind. Hence, for  $z'$  such that  $w(z') < \frac{1-R_L^{-1}\theta(1-\delta)}{R_L^{-1}\theta(1-\delta)} b_L$ , long-term debt will not affect the firm's choices. So,  $\gamma_l(w(z'), b_L, z', 0) = 0$  for sure. The cut-off level,  $\bar{z}$  is defined by  $w(\bar{z}) = \frac{1-R_L^{-1}\theta(1-\delta)}{R_L^{-1}\theta(1-\delta)} b_L$ .

Now I start to characterize the firm's optimal policy relation to its current state.

**Proposition 2.** *There exists a state-contingent threshold lever of net worth, above which the firm pays dividends. Firms with net worth above the threshold make the same capital, debt, and end period net worth decision: (i)  $\exists \bar{w}(b_L^-, z, s)$  s.t.  $\forall w \leq \bar{w}$ ,  $d = 0$  and  $\forall w > \bar{w}$ ,  $\mu = 1$ ,  $d > 0$ . (ii) Also,  $\forall w > \bar{w}(b_L^-, z, s)$ ,  $[d_o, k_o, b_{L,o}, b_{S,o} w_o(z')] = [w - \bar{w}(z), \bar{k}_o, \bar{b}_{L,o}, \bar{b}_{S,o} \bar{w}_o(z')]$  where  $[\bar{w}(z), \bar{k}_o, \bar{b}_{L,o}, \bar{b}_{S,o} \bar{w}_o(z')]$  attains  $V(\bar{w}(b_L^-, z, s), b_L^-, z, s)$ . (iii) Finally,  $x_o$  is unique for all  $w, b_L^-, z, s$ .*

As typical in models with decreasing return to scale and collateral constraint, the firm optimally chooses to invest all resource in capital when net worth is low. As the firm accumulates more net worth, it becomes less constrained since the marginal return on capital follows. Eventually, the firm starts to pay out dividend when it has sufficiently high net worth and is operating with a high level of capital. From that point on, any additional resource will have a marginal value of one. Hence, when net worth is sufficiently high, the optimal choices for capital, debts, and end period net worth are fixed and the firm pays out all additional resource as dividends.

**Result 3.** *Under the stationary distribution, firms with sufficient net worth never reduce their long-term debt level when refinancing is difficult. (i.e. when  $s = 0$  and  $\frac{R_L^{-1}\theta(1-\delta)}{1-R_L^{-1}\theta(1-\delta)}w > b_L^-$ ,  $b_L = b_L^- \forall (w, b_L^-, z, 0)$ )*

The Result 3 states that the firm is always constrained by the amount of existing long-term debt it has as long as it has sufficient net worth to support its current level of long-term debt. The intuition is as follows.

The firm can increase long-term debt only when it has access to external market. Since using more long-term debt hinders the accumulation of net worth for future and the firm value is always increasing in net worth, the firm that currently has access to external market will not choose to borrow with so much long-term debt such that the firm will have excess long-term debt next period when it has no access to external market (that is when  $s' = 0$ ). Therefore, under the stationary distribution, when the firm has no access to external market, firm will always try to use as much long-term debt as it can unless its net worth is insufficient to support its current level of long-term debt (that is when  $\frac{R_L^{-1}\theta(1-\delta)}{1-R_L^{-1}\theta(1-\delta)}w < b_L^-$ ).

When the firm does not have sufficient net worth compared to its old long-term debt (that is when  $\frac{R_L^{-1}\theta(1-\delta)}{1-R_L^{-1}\theta(1-\delta)}w < b_L^-$ ), it will still be collateral constrained when without access to external market. In that case, the firm is forced to repay a fraction of the existing long-term debt so that the new reduced level of long-term debt is fully backed up by its new capital level. Hence, default never occurs as in the typical model with collateral constrained.

Due to the complexity of the dynamic model, from this point on, I only discuss model implications for i.i.d. productivity shock case. At the end, I will present results on persistent productivity shocks in a simplified 2-period model.

### 1.3 Capital and Dividend Policy:

The main focus of the chapter is centered around the idea that firms that finance their capital more through long-term debt do not have to downsize as much when refinancing is difficult because less debt repayment is due. Figure 1.1 illustrates this point. The figure shows the optimal capital level as a function of net worth for each of the following three cases: with external financing, without external financing but with high long-term debt, and without external financing but with low long-term debt. Following Proposition 2, for each case, the capital level is constant when net worth is above a threshold  $\bar{w}(b_L^-, s)$ . Below the threshold, capital level is increasing in net worth and dividends are zero. First notice that even when the firm has sufficiently large net worth and existing long-term debt, it will still choose a lower capital level when has no access to external market (from the figure, the flat portion of dotted blue line is higher than the flat portion of the solid red line). The reason is that, when without access to external market, the firm can not raise short-term debt which has a lower interest rate. Thus, the marginal cost of funding must be higher than that when the firm has access to external market. That means that the marginal benefit from production must also be lower. And due to strict concavity of the production function, the firm must operate at lower capital level when without access to external market no matter how much net worth and existing long-term debt it has. Second, when the firm has less existing long-term debt, it is forced to operate at a lower capital level (region where the red line is above the green line). Hence, when the firm is previously with access to external market, suddenly the financing shock occurs and the firm is now without access to external market, it will have to downsize to a greater degree since it has used less long-term debt previously.

This fluctuation in capital level is costly to the firm because when the production technology is strictly concave, the firm sometimes has to pass on high returns on cap-



ital. However, the firm may optimally choose not to smooth the fluctuation because it is so constrained previously.

Finally, Figure 1.1 also shows that, when has no access to external market, the firm postpones dividend when it has low level of existing long-term debt. Indeed, the following proposition summarizes the observation.

**Proposition 4.** *When financing constrained, firms with more long-term debt pay dividends at a lower threshold. (i.e.  $\bar{w}(\tilde{b}_L^-, 0) > \bar{w}(b_L^-, 0)$ ,  $\forall \tilde{b}_L^- < b_L^-$ )*

Firms with more existing long-term debt pay out earlier for two reasons. First, when firms experience financing shocks, they can keep a higher capital level if they have more long-term debt from previous period because those debt are not due. In fact, as previously mentioned, those firms do operate with more capital and use more long-term debt currently (see Result 3). Second and consequently, they need less internal net worth to support the same level of capital next period when financing shocks occur. Hence, they optimally pay out the current extra resource as dividend.

As seen from Figure 1.1, the firm starts paying dividend when it is able to sustain a certain level of capital and its marginal return on capital is sufficiently low. Since the firm's current long-term debt choice can affect the fluctuation in capital level in future and since the firm starts to pay out when capital level is sufficiently high, the dividend dynamics will be affected by the firm's debt maturity choices. In particular, let's consider a firm which has sufficiently high net worth and pays dividends currently. Next period, if the financing shock occurs, depending on the realization of the production, either the firm has sufficient net worth so that it can keep operating at a sufficiently high capital level and paying a dividend or it has to defer dividend and put more net worth into financing capital investment. When the realization of the productivity and hence net worth are very low, the firm will have to downsize by selling its capital to repay debt and interests.

If the firm chose to raise more long-term debt previously, it will not have to downsize as much when financing shock occurs next period. Also, the firm can still pay out dividends in more states (productivity states) since in those states it can still operate at a sufficiently higher capital level. On the other hand, if the firm raises little long-term debt previously, it will have to downsize more and it has to omit dividends in more states (productivity states). To illustrate this point, I keep all other parameters the same and vary the probability of financing shock,  $p$ , as a comparative statics exercise. The results are presented in Figures 1.2 and 1.3. In Figure 1.2, each line shows the optimal next period net worth as a function of current next for a particular existing long-term debt and financing state,  $(b_L^-, s)$ . The point at which the line turns flat is the cut-off level of net worth at which the firm starts to pay dividends. The interval  $[w_L(\tilde{b}_L^-), w_H(\tilde{b}_L^-)]$  represents the region in which a firm with existing long-term debt,  $\tilde{b}_L^-$ , would pay dividend if it has access to external financing but would not pay dividend otherwise. However, if the firm has existing long-term debt,  $\hat{b}_L^- > \tilde{b}_L^-$ , instead, the corresponding region shrinks to  $[w_L(\hat{b}_L^-), w_H(\hat{b}_L^-)] \subset [w_L(\tilde{b}_L^-), w_H(\tilde{b}_L^-)]$ .

In Figure 1.3, I present the same result in the net worth and existing long-term debt plane. The solid parts of the two blue lines are the locus of different levels of possible next period net worth for two different levels of existing long-term debt. The segments  $[w_L(\tilde{b}_L^-), w_H(\tilde{b}_L^-)]$  and  $[w_L(\hat{b}_L^-), w_H(\hat{b}_L^-)]$  are the same as in Figure 1.2. The shaded region is where firms with different levels of existing long-term debt would pay dividends only if they have access to external market. Since the dividend paying cutoff  $\bar{w}(b_L^-, s)$  is constant when the firm has access to external market and decreasing in  $b_L^-$  when financing shock occurs, the segment  $[w_L(b_L^-), w_H(b_L^-)]$  shortens as the firm has more and more existing long-term debt. This means that the firm has to omit dividends in fewer productivity states when financing shock occurs if it managed to raise more long-term debt previously.

## 1.4 Short-term v.s. Long-term debt

In this Section, I describe the optimal short-term and long-term debt decisions. Due to the complex nature of the dynamic model, I only give some intuitions and show some numerical results.

### 1.4.1 *Cash as negative short-term debt*

Recently, many papers point out the fact that cooperations will save cash in case they will have difficulty raising new capital in future. My model suggests that firms can also use debt maturity management to help them endure times when financing condition is severe. In particular, when are able to adjust their debt structure, firms can control how much debt repayment due at the end of the period. When they expect to experience financing difficulties, they reduce the amount due at period end. Hence, firm's cash and debt maturity decisions may be naturally linked. As illustrated in my model, cash is equivalent to negative short-term debt. Therefore, only the net short-term borrowing matters.

Of course there are other reasons for why firms use short-term debt and for why firms hold cash reserves. Hence not all cash management activities can be viewed as substitutes for short-term debt changes. However, based on the newly developed theories on financing shock and cooperate cash savings, I believe that cash savings are to a large extent driven by the firms' concerns about future financing prospects. Also, the rollover risk associated with short-term financing is well recognized. Hence, firms' cash management and debt maturity decisions should be interwind. Most existing empirical papers on firms' debt maturity choices never control for cash. It is interesting to know whether their results could be overturned when properly taking into account the connection between cash and short-term debt. Also, I think that to what extent cash management affects the relation between firms' debt maturity and characteristics needs to be empirically investigated.

#### 1.4.2 Optimal long-term debt to capital ratio

Long-term debt level is in general increasing in net worth. The rationale is the following. Let's fix a given level of optimal long-term debt choice. When current net worth increases, capital level increases and next period net worth increases. Thus, there will be less next period states in which the firm will be collateral constrained given the long-term debt choice. And there will be more states in which additional long-term debt is desirable. Therefore, the firm raises the optimal long-term level when net worth increases. However, proportionally we observe the opposite pattern which is summarized in the next result.

**Result 5.** *Under the stationary distribution, firm with more net worth chooses a lower long-term debt to capital ratio. (i.e.  $\frac{b_L}{k}$  is decreasing in  $w$  for  $w \in [w_l, w_h]$ .)*

Due to the strictly concavity of the production function (Assumption 1), when net worth is low, the firm is more likely to have a next period net worth that is higher than the current net worth. For example, when the firm's net worth is lower than the lowest net worth of ergodic distribution of net worth  $w_l$ , its net worth is guaranteed to be higher in all states next period. This means it can support a higher level of debt next period. Therefore, initially in order to secure financing for the no access to external market state, the firm has to raise large amount of long-term debt compared to the relatively low level of net worth in current period. As current net worth increases, on average, the ratio of next period net worth to current net worth drop significantly. That is as the firm accumulates more net worth, its future net worth will drop relative to its current net worth in more states next period. Especially in some low productivity states next period, the firm will be forced to repay a fraction of the long-term debt such that the new level of long-term debt is fully collateralized by the firm's capital. Thus, additional long-term debt does not provide any benefit for those states. Hence, the firm finances less fraction of the capital using long-term

debt in the current period.

Therefore, even though in terms of level, the firm still borrows more long-term debt when it has more net worth, in proportion to current capital, the firm is borrowing less long-term debt. As illustrated in the left graph in Figure 1.4, the firm uses proportionally less and less long-term debt as net worth increases. Some papers documented that the long-term debt to total debt ratio increases with firm size. Others have found that the ratio increases with firm size for most the cases but decreases with size for very large firms. My model suggests that when the firm is able to adjust its debt structure, the optimal long-term debt to capital ratio is decreasing in net worth.

One undesirable pattern delivered from the model is the fact that firm with low net worth saves by lending out in short-term. This is shown in the right graph of Figure 1.4 and summarized as follows.

**Result 6.** *When the probability of financing shock is high, firms with low net worth choose to save through short-term debt. (i.e. for small  $w$ ,  $b_S(w, 1) < 0$ )*

Once again, when the firm has low net worth, its next period net worth is likely to be higher than that of today. Hence, the collateral constraint for next period is relaxed and the firm is allowed to borrow more in the absence of financing shock. Therefore, not having access to external market is very costly for the firm next period. Consequently, it will use a lot of long-term debt today in case it won't be able to borrow next period. In fact, when current net worth is low, the firm will save with short-term debt as collateral in order to borrow more long-term debt in current period. However, in the present of a fixed cost for long-term debt, firms with low net worth will not use long-term debt to raise capital.

## 1.5 Fixed Cost of Long-term Debt Issuance

Now I reformulate the firm's problem in a more general form with fixed cost of issuing new debt. In particular, I assume that depending on the financing state, the firm needs to pay a fixed cost  $\phi_S(s)$  and  $\phi_L(s)$  for issuing new short-term and long-term debt. In practice, the issuing cost is higher for long-term debt and both issuing costs are higher in bad financing state. Hence, the fixed costs have the following relations:  $\phi_S(s^+) < \phi_S(s^-)$ ,  $\phi_L(s^+) < \phi_L(s^-)$ ,  $\forall s^+ > s^-$  and  $\phi_S(s) \leq \phi_L(s)$ ,  $\forall s \in S$ . Hence, the baseline model can be formulated by letting  $\phi_S(0) = \phi_L(0) = \infty$  and  $\phi_S(1) = \phi_L(1) = 0$ .

To explore the effect of fixed cost on long-term debt to firm maturity decision, I keep all baseline setup the same except assuming  $\phi_L(1) > 0$ . Certainly, if  $\phi_L(1)$  is too high, then the firm will never use any long-term debt. Hence, I am only interested in the case where  $\phi_L(1)$  is such that some firms will raise additional long-term debt when with access to external market under the stationary distribution.

In the presence of fixed cost, firms with low net worth will only raise short-term debt. The intuition is that when the firm has low net worth, it is very constrained hence paying the fixed cost is very costly. Thus, the firm uses cheaper short-term debt instead and tries to accumulate more net worth. As net worth increases, the firm becomes less constrained and cares less about the current fixed cost. As shown in the left graph of Figure 1.5, the firm does not raise new long-term debt when it has low net worth (i.e. the optimal choice of  $b_L$  is the same as  $b_L^-$ ). Instead, the firm only raises short-term debt as shown in the right graph of Figure 1.5. Then, as the firm's current net worth increases, the ratio between the firm's next period marginal value of net worth and current marginal value of net worth decreases. That is, the current resource is not much more valuable than future resource. Therefore, the firm pays the fixed cost and raises large amount of long-term debt so that next period it will have

more resource from long-term when the financing shock hits. This corresponds to Region A in Figure 1.6. Also, shown in Figure 1.5, this region is where the long-term debt jumps up and the short-term debt jumps down. In fact, as shown in Region C of Figure 1.6, there is a threshold level of net worth  $\hat{w}(b_L^-)$  below which the firm will choose to not to raise new long-term even when it has access to external market. This threshold depends on the firm's existing long-term debt level. The higher the existing long-term debt the higher the threshold. The rationale is related to the trade-off I will discuss next.

Basically, the fixed cost introduces additional concerns for the firms that currently have some existing long-term debt. More specifically, if the level of existing long-term debt is high, the firm will not want to pay the fixed cost to raise additional long-term debt. Since the benefit of long-term debt is somewhat proportional to the level of long-term debt raised, there are economies of scale in using long-term debt. Hence, only when the amount of new capital raised is sufficiently large, the firm feels justified to pay the fixed cost. This corresponds to Region B in Figure 1.6 and can be seen in region where optimal long-term debt choices equal to its existing level in Figure 1.5. Hence, when both net worth and existing long-term debt are high, the firm also chooses not to raise new long-term debt. Finally, if the firm has high levels of existing long-term debt but got hit by a low productivity shock, its net worth drops and will have to downsize. However, now the firm will no longer choose to pay any of the long-term debt, instead it saves the proceeds from capital sale by lending in short-term debt. Then, it uses that short-term debt as collateral to support its high level of long-term debt. The reason is that if the firm reduces long-term debt now, it may have to raise additional long-term debt again. Hence, to reduce the chance that it will have to pay the fixed cost again in the future, the firm chooses to defease the existing long-term debt in order to keep all of it.

## 1.6 Current v.s. Future Investments and Debt Maturity

For simplicity, I abstract from net worth effects for now and assume that the firm has a linear production function. That is,  $f_t(k_t) = k_t$  and  $f'_t(k_t) = 1$ .

To facilitate the interpretation of the solution, I define the following terms. As in Rampini and Vishwanathan (2010), I define down payment as the minimum amount that the firm has to put down per unit of capital. In the case, it is one minus the proportion borrowed with debt. Hence, the down payments with short-term financing and with long-term financing are respectively,  $p = 1 - R^{-1}\theta(1 - \delta)$  and  $p_L = 1 - R_L^{-1}\theta(1 - \delta)$ . Further, the reciprocals  $\frac{1}{p}$  and  $\frac{1}{p_L}$  are the maximum leverages through short-term and long-term financing. Notice that since long-term debt is more expensive (i.e.  $R_L > R$ ), short-term debt requires less down payment and offers higher leverage. Later on, we will see that the firm trades off this benefit of short-term debt with the cost of underinvestment. Now, I denote the net fully levered average return on capital with  $Ret(A_t) = A_t + (1 - \delta)(1 - \theta)$ , the subscribe  $t$  denotes time. Hence, if the firm only uses short-term debt, the net fully levered average return on internal funds (net worth) is  $\frac{Ret(A_t)}{p}$ . If the firm only uses long-term debt, the net fully levered average return on internal funds is  $\frac{Ret(A_t)}{p_L}$ . Finally, I denote the ordinary gross return on capital with  $\hat{Ret}(A_t) = A_t + (1 - \delta)$ .

Due to the linear objective at time 1, the firm either pays out all net worth as dividends or invests as much as it can depending on the average productivity in time 2. If firm pays out all net worth at time 1, then at time 0 the firm is facing a one period problem. It only needs to care about end of period net worth. Long-term debt is never used at time 0 since it demands a higher return but provides no additional benefit over short-term debt.

In order to focus on more interesting cases, I assume that the capital is sufficiently productive at time 2, that is:



**Assumption 3.**  $E_1 A_2 + (1 - \delta) > \frac{1}{\beta}, \forall A_1$ .

With sufficiently high time 2 productivity, the firm invests all resources in capital and pays out nothing at time 1. Moreover, if it has access to external financing ( $s = 1$ ), it will retire all its current long-term debt and borrow only short-term debt until it runs out of collateral. However, if the firm has no access to external financing, it will be constrained either by its collateral or by its long-term debt holding depending on the relation between its beginning long-term debt holding and net worth at time 1. Hence, the firm's time 1 decisions are straight forward.

**Lemma 7.** *Under Assumption 3, the firm's decisions at time 1 can be characterized as follow:*

- *If the firm has access to external financing ( $s = 1$ ), then*

$$d_1 = 0, k_2 = \frac{w_1}{p}, b_2 = (\frac{1}{p} - 1)w_1, b_2^L = 0, w_2(A_2) = \frac{Ret_2}{p}w_1.$$

- *If it is without external financing ( $s = 0$ ) and poorly capitalized (i.e.  $b_1^L \geq (\frac{1}{p_L} - 1)w_1$ ), then*

$$d_1 = 0, k_2 = \frac{w_1}{p_L}, b_2 = 0, b_2^L = (\frac{1}{p_L} - 1)w_1, w_2(A_2) = \frac{Ret_2}{p_L}w_1.$$

- *If it is without external financing ( $s = 0$ ) and well capitalized (i.e.  $b_1^L < (\frac{1}{p_L} - 1)w_1$ ), then*

$$d_1 = 0, k_2 = w_1 + b_1^L, b_2^L = b_1^L, b_2 = 0, w_2(A_2) = \hat{Ret}_2(w_1 + b_1^L) - R_L b_1^L.$$

where  $w_1$  and  $b_1^L$  are the time 1 beginning net worth and long-term debt holding. And  $d_1, k_2, b_2, b_2^L$  are respectively the dividends payment, the capital level, short-term debt holding, and long-term debt holding chosen at time 1. Finally,  $w_2(A_2)$  is the end of period payout in each state of time 2 with productivity  $A_2$ .

Lemma 7 classifies three type of states at time 1 and characterizes the optimal choices and payoff for each type. For convenience, through out the chapter, I call the states in which the firm's time 1 decision is constrained by its net worth the “collateral-constrained” states. I call the states in which the firm's time 1 decision is constrained by its long-term debt holding the “debt-constrained” states. Since the beginning long-term debt holding  $b_1^L$  is decided at time 0 and because the time 1 net worth is one-to-one and increasing in time productivity 1, I can define and solve for the cut-off value for determining the last two types of states.

**Lemma 8.** *There exists a cut-off value of time 1 productivity,  $\underline{A}$ , defined by:*

$$\underline{A} \equiv \left\{ A_1 : w_1(A_1) = \frac{b_1^L p_L}{1 - p_L} \right\}. \quad (1.12)$$

*such that for all states above  $\underline{A}$ , the firm is debt constrained and for all states below  $\underline{A}$ , the firm is collateral constrained.*

At time 0 the firm chooses  $d_0, k_1, b_1, b_1^L, w_1(A_1)$ , to maximize the payout over the two periods. The choices of  $b_1^L$  and  $w_1(A_1)$  will determine which states are debt-constrained and which are collateral-constrained. And at time 0, the firm knows exactly what it will do in each state at time 1. Also, the return per capital only differs between different types of states. Under parameterizations such that the collateral constraint is binding at time 0, I can reformulate the firm's time 0 problem in terms of the ratio of optimal long-term debt to time 0 net worth,  $\alpha = \frac{b_1^L}{w_0}$ . And it can be shown that the optimal long-term debt to net worth  $\alpha$ , is pinned down by the following FOC:

$$\begin{aligned} & \left\{ (1 - \pi) \mathbb{E} \frac{Ret(A_2)}{p} Ret(A_1) + \pi \Pr[A_1 \leq \underline{A}] \mathbb{E}_{A_1 \leq \underline{A}} \frac{Ret(A_2)}{p_L} Ret(A_1) \right. \\ & \left. + \pi \Pr[A_1 > \underline{A}] \mathbb{E}_{A_1 > \underline{A}} \hat{Ret}(A_2) Ret(A_1) \right\} \frac{(p_L - p)}{p_L p} = \pi \Pr[A_1 > \underline{A}] \frac{1 - p_L}{p_L} \mathbb{E}_{A_1 > \underline{A}} [\hat{Ret}(A_2) - R_L] \end{aligned}$$

In essence, the firm chooses that optimal ratio such that the marginal cost is equal to the marginal benefit of long-term debt. The left hand side of the equation represents the marginal cost of long-term debt. It is the loss on the return on net worth, which is the sum in the curly bracket, times the proportional reduction in time 1 net worth. The right hand side of the equation represents the marginal benefit of long-term debt. It is the gain from the difference between return on capital and long-term debt interest rate times the proportional increase in long-term debt. So the firm balances the loss from net worth reduction and the benefit from having more long-term debt at time 1.

The following three propositions describe the effects of productivity prospects on the firm's maturity decisions:

**Proposition 9.** *Firms with more persistent productivities proportionally use more long-term debt (i.e.  $\alpha$  increases with  $\text{corr}[A_1, A_2]$ .)*

**Proposition 10.** *When productivity shocks are independent, firms with better average period 2 productivity proportionally use more long-term debt (i.e. When  $A_1, A_2$  are independent,  $\alpha$  increases with  $EA_2$ .)*

Proposition 10 states that if the firm has better investment opportunities in the distant future (at time 2), it will choose to use more long-term debt now. The reason is that if the firm currently chooses an interior long-term debt level ( $\alpha > 0$ ), it must be that the proportional increase in long-term debt  $b_L^-$  is a lot bigger than the proportional decrease in time 1 net worth  $w_1(z')$  caused by an increase in  $\alpha$ . Hence, if the return on capital for time 2 improves, the marginal benefit of long-term debt raises more than the marginal benefit of time 1 net worth. Similarly, since we consider a case in which the time 1 productivity is sufficiently high on average, an increase in the correlation of the two shocks means better future productivities

(at time 2). Therefore, the firm also increases its use of long-term debt when the correlation between time 1 and time 2 productivities increases.

The results have some similarities to the project and debt maturity matching story. Basically, if most of the returns will come from future periods, the firm is more afraid from being cutoff of fund supply and hence won't have enough capital to take advantage of those good returns. Hence, it starts to use more long-term debt now to secure the access of fund. On the contrary, if the firm high current productivity, it will do the opposite as summarized in Proposition 11.

**Proposition 11.** *When probability of financing shock is low, firms with higher current productivities proportionally use less long-term debt (i.e. for low  $\pi$ ,  $\alpha$  decreases when  $A_1$  increases state by state.)*

When the current productivity rises, there are two effects. First, the firm can gain more from the higher leverage offered by the short-term financing. Second, because the firm now will have more net worth next period on average, it can sustain higher leverage next period. Hence, a market freeze will hurt the firm more. This second effect makes the firm more willing to use long-term debt now, but it depends on the probability of market freezes. When the probability,  $\pi$ , is low, the firm cares more about capitalizing the current gains with more short-term financing. The proposition suggests that firms that use relatively less long-term debt are the ones that have good current investment opportunities. However, firms with less long-term debt will have to downsize more when experiencing financing shocks in future. When the productivity shocks are persistent, this means that at time 1 when financing condition is severe, capital is inefficiently deployed across firms with different investment opportunities. The ones with high return on capital have relatively less capital. Whereas the ones with low return on capital have relatively more capital since they used more long-term debt previously.

In practice, we do observe that firms with more growth options finance with more short-term debt. Empirically, people have found that firms with higher marginal  $Q$  and higher sales growth tend to have a larger fraction of short-term debt. Traditionally this pattern is attributed to Myers' debt overhang story. More specifically, the firm with more future investment opportunities wants to have its debt mature before the investment decision so that there will not be any incentive distortions between the equity and debt holders. My model provides an alternative explanation. In the presence of both collateral and financing constraints, firms are trading off net worth with long-term debt. When the probability of experiencing financing shocks is low, firms with better current investment opportunities concern more about accumulating net worth and are willing to risk having to downsize more when a financing shock occurs.

**Proposition 12.** *When financially constrained, firms use more short-term financing as the cost of borrowing rises. (i.e. When  $\frac{\theta(1-\delta)}{R}$  is low,  $\alpha$  decreases from a proportional increase in both  $R$  and  $R_L$ )*

Last but not least, the model suggests that more constrained firms use more short-term financing. This conclusion is profound but intuitive. The more constrained firms borrow less from the financial markets. Hence, they rely more on their internal net worth to finance most of their investments. As a result, they have less incentive to hedge for future financing shocks. At the same time, since net worth is so important, they want to build their net worth faster by investing more today. Therefore, they use more short-term financing to increase leverage and production. This conclusion is important because it improves our understanding of the relation between financial constraints and debt maturity. Prior research blamed firms for holding excessive amounts of short-term debt under the belief that short-term financing made firms more vulnerable to financial shocks, and frequent repayments due to a shorter-term debt structure made firms more constrained. My model explains that firms borrow

short-term debt precisely because they are constrained. Thus, it is financial constraints that cause more short-term financing, not the other way around. In the next section, I use a natural experiment to empirically test for this direction of causality.

## 1.7 Empirical Testing

In this section, I verify my model predictions and explore its potential in three steps. First, I test whether more constrained firms use more short-term financing. Second, I propose a measure for financial constraints based on my model and examine the performs of my measure. Finally, I show that the proposed measure is consistent with the natural experiment I use for some of the empirical tests.

### *1.7.1 Financial constraints and debt maturity*

In the literature, measuring financial constraint is very difficult. Simply showing correlation between debt maturity and some exiting measures of financial constraints is not sufficient because the measures themselves may not reflect financial constraints well. Plus, even if the measures for financial constraints are precise, simple correlation still does not show causality due to potential endogeneity and simultaneity. Therefore, I use a quasi-natural experiment to show that financial constraints cause firms to use short-term debt.

Previous literature has shown that increase in competition leads to higher cost of debt, lower profitability, and less research and development for the firms that experience that increasing competition (Valta (2012), Hou and Robinson (2006), and Fresard and Valta (2013)). Hence, I think that firms who experience more competition are more constrained. Therefore, I use exogenous shocks to competition as exogenous shocks to firm financial constraints.

My main data consist of Compustat and international trade data from Peter Schott's web site. The sample period is from 1989 to 2005. I exclude firms in the

financial (SIC 6000-7000) and utility (SIC 4400-5000) industries. Also, the sample is restricted to those industries (mainly manufacturing) present in the international trade data. I delete all observations with total assets less than five million since the very small firms tend to behave abnormally. I also require the observations to have sufficient information for calculation control variables such as marginal Q, leverage, asset maturity, etc. Main variables are defined in Table 1.8 in Appendix E. The exogenous shocks to competition are define as follows.

I collapse the data to industry-years to compute the ad valorem tariff rate as the sum of duties charged divided by the dutiable import value:

$$\text{Ad Valorem Tariff Rate}_{j,t} = \frac{\sum_{k=1}^{N_j} \text{Duties}_{k,j,t}}{\sum_{k=1}^{N_j} \text{Dutiable Import Value}_{k,j,t}} \quad (1.13)$$

where  $k$  indexes countries,  $j$  indexes industries, and  $t$  indexes time. I then compute the change in the ad valorem tariff rate for each industry-year. Next, I compute the median tariff rate change by industry. Following Valta (2012), an industry-year has a “competitive shock” if the absolute value of the largest tariff rate reduction is greater than three times the absolute value of the median tariff rate change for that industry. Industries that experience competitive shocks are said to be treated. I exclude tariff rate reductions that preceded and followed by equivalently large increases in tariff rates. The summary statistics for all variables are shown in Table 1.1.

From those calculations, I define for each industry the indicator variable  $\text{Postreduction}_{j,t}$ , which equals one if the tariff rate reduction has occurred in industry  $j$  by time  $t$ . I identify 34 large tariff rate reductions in 34 three digit SIC code industries between 1989 and 2005. Then I estimate the following difference-in-differences specification:

$$\text{Maturity}_{i,j,t} = \alpha_i + \alpha_t + \lambda(\text{Postreduction}_{j,t}) + \beta' X_{i,t} + \epsilon_{i,j,t} \quad (1.14)$$

where  $\alpha_i$  are firm fixed effects;  $\alpha_t$  are year fixed effects;  $\text{Postreduction}_{j,t}$  is an

indicator variable equal to 1 if industry  $j$  has been treated by year  $t$ , and zero otherwise;  $Maturity_{i,j,t}$  are the fraction of debt which matures in more than 3 year. It is a measure for the proportion of long-term debt for each firm-year observation;  $X_{i,t}$  is a set of control variables for each firm-year observation. I use white robust procedure cluster standard errors at the firm level. Hence, my estimates account for heteroskedasticity and serial correlation in the error term. The results are present in Table 1.2. The slope on the dummy variable  $Postreduction_{j,t}$  is negative and significant. This suggests that compared to the control firms, the treated firms reduced their debt maturity after the shocks that raised competition. This reduction is both economically and statistically significant. As shown in Table 1.1, the average fraction of long-term debt is 0.42. After the shocks to competition, the treated firms reduced that fraction by about 7.9%. To make this number more accessible, let's assume that all debts maturing within 3 years have an average maturity of 2 years and all debts maturing after 3 years have an average maturity of 15 years. The 7.9% reduction corresponds to a reduction in debt maturity from 7.5 years to 7 years.<sup>4</sup> If we believe that higher competition leads to a more constrained environment for the firms within the industries, the result suggests that more constrained firms use more short-term debt.

### 1.7.2 Investment-debt sensitivity

Since my models link debt maturity to financial constraints, it can be used to develop a good measure for financial constraints. Previous attempts in the literature in constructing a proxy of financial constraint are somewhat unsuccessful. I think that it is very important to have a good way to assess how constrained firms are. By far, the most established one is Whited Wu index (Whited and Wu 2006). I will use the

---

<sup>4</sup>The percentage reduction in fraction of long-term debt is obtained by  $.033/.42 \approx 7.9\%$ , where .42 is the mean of "Maturity3" in Table 1.1 and .033 is the absolute value of the coefficient on "Postreduction" in column (1) of Table 1.2. The average debt maturity is obtained by  $.42 * 15 + (1 - .42) * 2 \approx 7.5$ . Finally, the debt maturity after the shock is obtained by  $(.42 - .033) * 15 + (1 - .42 + .033) * 2 \approx 7$ .



index and some other classifications of financial constraint to show that measure does proxy for financial constraints.

From the model, we see that when the firm takes on large amounts of debt previously and has to pay it back, it become very constrained so that it has to forego good investment opportunities. Because of this negative impact of debt repayment to investment, I think the sensitivity from investment to debt repayment is a good measure for financial constraints. Suppose that a firm is totally unconstrained and it can always borrow as much as it wants. Then no matter how much debt it has to repay, its investment decision is not affected because it simply borrows more debt to satisfy both the investment and repayment needs. On the other hand, suppose a firm is very constrained and it can not borrow any new debt. Then when this firm has lots of debt repayment, it has to decrease investment dramatically in order to make debt repayments. Therefore, I suspect that the sensitivity of investment to debt repayment is negative and significant. Furthermore, more constrained firms should have more negative sensitivities of investment to debt repayment.

In order to find the sensitivity of investment to debt repayment, I perform the following regression:

$$\frac{I_{it}}{K_{it-1}} = \alpha_i + \alpha_t + \beta_1 \times q_{it-1} + \beta_2 \times \frac{CF_{it}}{K_{it-1}} + \beta_3 \times \frac{DR_{it}}{K_{it-1}} + \epsilon_{it} \quad (1.15)$$

where  $\alpha_i$  are firm fixed effects;  $\alpha_t$  are year fixed effects;  $\frac{I_{it}}{K_{it-1}}$  are proxy for investment in year  $t$  for firm  $i$ ;  $\frac{CF_{it}}{K_{it-1}}$  are proxy for cash flow in year  $t$  for firm  $i$ ;  $\frac{DR_{it}}{K_{it-1}}$  are proxy for debt repayment in year  $t$  for firm  $i$ ; I use white robust procedure and cluster the standard errors at the firm level. For this exercise, since observations are not constrained by the availability of international trade data, I use all Compustat data from 1967 to 2011.

The results for regressions using full panel are shown in Table 1.3. Indeed, we see that the coefficients for  $\frac{DR_{it}}{K_{it-1}}$  are negative and significant in both the regressions

with and without additional control variables. The coefficient  $-.264$  in Column (Basic) suggests that one standard deviation increase in debt repayment leads to about 7.8% decrease in investment rate,  $\frac{I_{it}}{K_{it-1}}$ . Hence, debt repayment indeed has a significant effect on investment. Next, I examine whether the coefficients for  $\frac{DR_{it}}{K_{it-1}}$  (the investment-debt sensitivity) are more negative for more constrained firms. Since the Whited Wu index, firm size, and dividend rates are so far the best proxy for financial constraints, I split the data into sub-samples based on those proxy for financial constraints. Then I run the regression for each sub-sample and see if the coefficients for  $\frac{DR_{it}}{K_{it-1}}$  (the investment-debt sensitivity) are more negative for the more constrained sub-samples.

Table 1.4 shows the results for regressions on sub-samples based on Whited Wu index. As we move from the least to most constrained sub-samples, the coefficients for  $\frac{DR_{it}}{K_{it-1}}$  indeed become more negative and significant. For example, the sensitivity is  $-.352$  in Column (5) but is only  $-0.071$  in Column (1). This means that for the most constrained firms (Column (5)), one standard deviation increase in debt repayment leads to a 10.9% reduction in investment rate. However, the same increase only leads to a 2.1% reduction in investment rate for the least constrained firms. So the effect of debt repayment on investment is substantially bigger for the more constrained firms. The patterns are the same across sub-samples based on size and dividend rate (Table 1.5, Table 1.6). Therefore, I conclude that more constrained firms have more negative investment-debt sensitivity.

In unreported robustness tests over subperiods, the investment-debt sensitivity remains reliable for all time periods from 1967 to 2011. Moreover, although the amount of debt repayment due next year may be related to previous financing and investment decisions, it does not relate to future investment opportunities directly. Therefore, I believe most of the effect of debt repayment on investment depends on how constrained the firm is. Hence, investment-debt sensitivity is unlikely to suffer

endogeneity problems as investment-cash flow sensitivity does.<sup>3</sup>

### 1.7.3 Investment-debt sensitivity and the natural experiment

With the new measure for financial constraint, I now check whether the increase in competition corresponds to increase in financial constraints. More specifically, I examine whether the investment-debt sensitivity for the treated firms becomes more negative after the increase in competition compared to the control firms. I conduct the following regression:

$$\begin{aligned} \frac{I_{i,j,t}}{K_{i,j,t-1}} = & \alpha_i + \alpha_t + \lambda_1 \frac{DR_{i,j,t}}{K_{i,j,t-1}} + \lambda_2 Post_{j,t} + \lambda_3 \left( \frac{DR_{i,j,t}}{K_{i,j,t-1}} \times Post_{j,t} \right) \\ & + \beta' X_{i,t} + \epsilon_{i,j,t} \end{aligned} \quad (1.16)$$

where  $\alpha_i$  are firm fixed effects;  $\alpha_t$  are year fixed effects;  $\frac{I_{it}}{K_{it-1}}$  are proxy for investment in year  $t$  for firm  $i$ ;  $\frac{DR_{it}}{K_{it-1}}$  are proxy for debt repayment in year  $t$  for firm  $i$ ;  $Post_{j,t}$  is an indicator variable equal to 1 if industry  $j$  has been treated by year  $t$ , and zero otherwise;  $\frac{DR_{i,j,t}}{K_{i,j,t-1}} \times Post_{j,t}$  is the interact between  $Post_{j,t}$  and  $\frac{DR_{it}}{K_{it-1}}$ ; This interaction term captures the change in investment-debt sensitivity for the treated group before and after the shock compared to that of the control group. Hence, I test to see whether coefficient for the term  $\frac{DR_{i,j,t}}{K_{i,j,t-1}} \times Post_{j,t}$  is significantly different from zero. In Table 1.7, we see that the coefficient for the term  $\frac{DR_{i,j,t}}{K_{i,j,t-1}} \times Post_{j,t}$  is negative and significant with or without additional controls. Therefore, the data suggest that the firms which experienced the increase in competition become more constrained after the shock.

## 1.8 Related Literature

One of the important elements of my model is the rollover risk caused by the market freezes. Throughout the literature on debt maturity, all papers have rollover risk

as the cost of short-term debt.<sup>5</sup> Some try to explain how capital markets break down. For example, Brunnermeier and Oehmke (2013) address financial institutions' maturity mismatch problem in an equilibrium frame work. In the model, short-term creditors may demand a higher face value on the new debt when their lending matures and needs to be rolled over in states where default is more likely. Long-term creditors bear the cost of increasing the face value. The bank (the borrower in their model) is unable to commit to a single maturity structure. It optimally chooses to use more short-term debt upon receiving interim signal regarding the probability of default. Consequently, every creditor prefers to shorten its lending maturity and long-term financing unravels. Focusing on the effects and implications of short-term financing, Acharya, Gales, and Yorulmazer (2011) demonstrate that when the arrival of good news is slower than the rate of rollover and when everyone is borrowing at short term, liquidity dries up quickly and a market freeze may occur despite the fact that the underlying asset quality is unchanged. This is due to the fact that, when a debt is rolled over frequently, little information is revealed between rollover dates. Thus, the difference in the debt capacities between high and low current states is big, even though the fundamental values are almost the same. Both those papers provide micro-foundation for the market freezes assumed in my model.

A variety of benefits of short-term financing have been introduced. For example, short-term financing reduces the underinvestment problem caused by debt overhang (Myers (1977); Childs, Mauer, and Ott (2005); Diamond and He (2012)). The idea is that if debt matures and is settled before the firm decides whether to take a new project, there will be no incentive distortion. In my model, short-term debt also causes underinvestment because the firm has to use resource to repay the debt. Especially, when it experiences difficult obtaining new finance, the firm has to scale down.

---

<sup>5</sup>All theory papers mentioned in this section deal with rollover problem of short-term debt. Other important papers include Leland (1994), Leland and Toft (1996), Leland (1998), He and Xiong (2012a 2012b), and He and Milbradt (2013)

However, short-term debt also helps to increase investment in current period since it is borrowed at relatively low cost. So the firm's main trade off is between current investment and future investments.

Segura and Suarez (2013) have a model that is similar to mine. In their model, short-term debt is relatively cheaper since it better accommodates preference shocks that investors may experience. However, when preference shocks occur, the lending supply shrinks dramatically. In deciding the optimal fraction of long-term and short-term debt to use, banks trade off the lower cost of borrowing of short-term debt with the higher cost when refinancing during systemic liquidity crisis. They conclude that the banks' optimal choice of long-term debt holding is inefficient. Government regulations, such as debt maturity limits, Pigovian taxes, and liquidity insurances, can make welfare improvements. Different from theirs, the main drive in the model is the productivity shocks. The firm makes real investment decisions rather than having a fix inflow every period.

In addition, short-term debt can be used as a disciplining device against agency problems. In particular, many consider the problem of "risk-shifting" (Barnea, Haugen, and Senbet (1980); Grossman and Hart (1982); Calomiris and Kahn (1991); Leland (1998); Diamond and Rajan (2001); Eisenbach (2013); Cheng and Milbradt (2012)). Others also build on the fact that short-term debt can help to prevent managers from "empire-building". With short-term debt, the creditors can decide whether to continue the project if managers fail to pay. Since managers want to continue negative NPV projects due to their private benefit, it is important for the creditors to take control before things go awry (Grossman and Hart (1982); Benm-  
elech (2006)). In my model, there is no incentive problem between the equity holders and the managers. The benefit of short-term debt is simply the low interest rate. In a dynamic environment, firms are more likely to take advantage of the low rate when they need capital the most.

Berglof et al. (1994) point out that when future returns are non-verifiable, short-term debt can help the firm commit not to repudiate in the future, hence increases the firm's credibility. They argue that the more non-verifiable the long-term returns, the larger shares of short-term debt are required. They predict that the use of short-term debt increases with the amount of outside finance. In my model, larger fraction of short-term debt usage also increases total capital borrowed due to higher leverage of short-term debt.

The setup and predictions of my model are consistent with most of the empirical findings. For example, Johnson (2003) documents that liquidity risk increases with shorter maturity. In my model, firms with larger proportion of liabilities due in short-term are more exposed to rollover risk and more vulnerable to market freeze.

Barclay and Smith (1995), Guedes and Opler (1996), Barclay, Marx, and Smith (2003) and Scherr and Hulburt (2001) provide strong evidence that corporate debt maturity is negatively associated with growth opportunities. They use current Tobin's Q as the measure for growth opportunities. However, Tobin's Q could simply be a proxy for next period return on capital. The model predicts that when next period return is higher than the returns in longer future, firms use more short-term debt. Importantly, my explanation is different from Myers' debt overhang story. In my model, firms use more short-term debt because they want to raise and invest more today to capture the high next period return.

Recently, a few start to investigate the relation between aggregate debt maturity and business cycle. Mian and Santos (2011), Broner et al. (2010), and Wang, Sun, and Lv (2010) provide evidence that the corporate debt maturity choice is pro-cyclical. In particular, Broner et al. (2010) show that short-term debt demands less risk premium in emerging economies. Also, the relative cost of long-term borrowing increases during crises. Hence, there is relatively more use of short-term debt during crises. Their results support my setup for the benefit of short-term debt. Mian and Santos (2011)

also show a general decline in the debt maturity from 1988 to 2010. According to my model, I suspect that the decline is driven by reduction of hardship for obtaining external finance due to improvement in searching technology and the development of intermediaries and the capital markets.

Finally, to investigate the relationship between debt maturity and the underlying assets, Benmelech et al. (2005, 2008) document that debt maturity increases with the redeployability of assets, and verify that asset salability is also positively related to debt maturity. For example when demand of the underlying assets is higher, firms can use more long-term debt. Furthermore, Julio, Kim, and Weisbach (2008) find that firms use more long-term bonds when financing investments in fixed assets and use more short-term bonds when financing investments in *R&D*. My model provides mixed conclusions on the relation between pledgeability and debt maturity. On one hand, higher pledgeability means the firm can lever even more with short-term debt relative to long-term debt. On the other hand, higher pledgeability implies a higher refinancing risk for constrained firms due to higher leverage. Therefore, more detailed and better designed empirical studies are needed to distinguish between the two effects and resolve the mixed results.

## 1.9 Conclusion

This chapter tries to rationalize firms' debt maturity decisions in the presence of financing frictions that prevent firms from raising new capital. Firms trade off between short-term and long-term debt based on the fact that long-term debt is more costly but needs to be refinanced less often. Since firms are currently constrained by their net worth, they do not borrow sufficient long-term debt as to fully smooth capital between good and bad financing states. As a consequence, firms' return on capital and marginal benefit of net worth fluctuate with the financing conditions. Dividends also fluctuate between good and bad financing states. In many cases, firms have to

omit dividends when they experience financing difficulties. Especially, when firms think they have higher chance of obtaining new external financing in future, they borrow more short-term debt and less long-term debt. In turn, they have to omit dividends in more states when the financing shock hits.

Moreover, with fixed costs, the model is consistent with the fact that small firms do not use long-term debt. It also explains why sometimes firms choose to save cash instead of paying down outstanding debt. Especially, when financing condition is severe, firms with large amount of existing long-term debt relative to their net worth would use cash savings as collateral to support their outstanding debt. In doing so, they pass on good investment opportunities and high returns on capital.

In a simplified two period version of the model, I explore the effects of current and future investment opportunities on firm's debt maturity choice. The model speaks to the pattern that firms with good investment opportunity use more short-term debt. It shows that those firms are willing to bear the risk of severely downsizing and passing on investment opportunity in future because they are more concerned about taking advantage of current investment opportunity and growing more net worth. Using a quasi-natural experiment, I verify that more constrained firms use more short-term debt. Finally, I propose investment-debt sensitivity as a new measure for financial constraints. The data confirm that more constrained firms are associated with more negative investment-debt sensitivities. Indeed, firm debt maturity choice is closed related to financial constraints and firm investment needs.



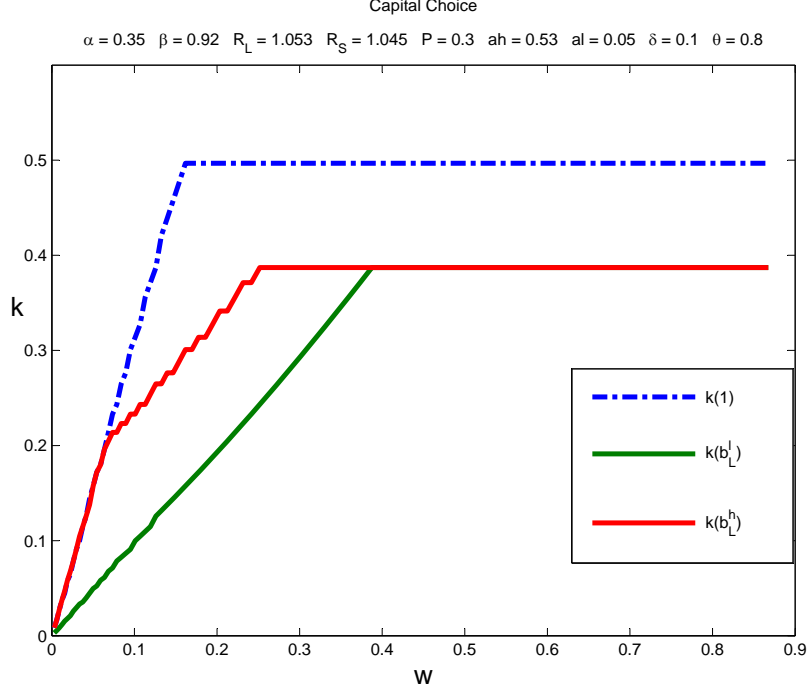


FIGURE 1.1: **Capital Choices.**

This figure shows the optimal capital choices as functions of net worth for three cases. The y-axis is capital level and x-axis is net worth level. The blue dotted line is the optimal capital policy for a firm which currently has access to the external market ( $s = 1$ ). The red solid line is the optimal capital policy for a firm which has no access to external market ( $s = 0$ ) but with a high existing long-term debt level  $b_L^h$ . Finally, the green solid line is the optimal capital policy for a firm which has no access to external market ( $s = 0$ ) but with a low existing long-term debt level  $b_L^l$ . The parameter values used in the numerical example are listed at the top of figure. I used  $f(k) = k^\alpha$  as the production function and  $A(z)$  is uniform distributed from 0.05 to 0.53.

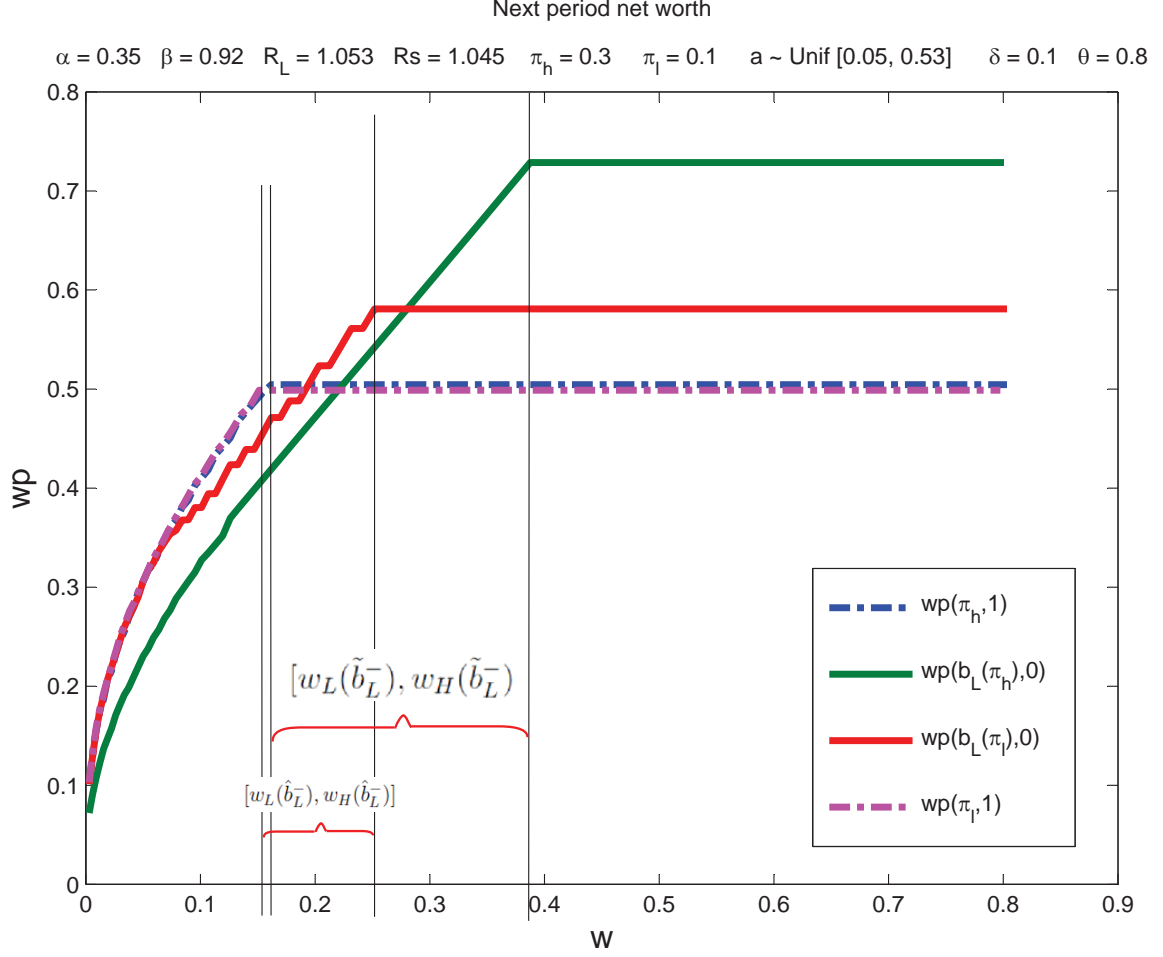


FIGURE 1.2: **Dividends Payouts.**

The figure shows two pairs of optimal next period net worth policies as functions of currently net worth for the highest productivity realization. The y-axis is next period net worth level and x-axis is current net worth level. The green and blue lines are policies for a firm in environment with low probability of experiencing financing shock ( $\pi = \pi_l$ ). In particular, the blue dotted line is the policy when the firm has access to the external market ( $s = 1$ ). The green solid line is the policy when the firm has no access to external market ( $s = 0$ ) but with existing long-term debt level  $b_L(\pi_l)$ .  $b_L(\pi_l)$  is the optimal long-term debt chosen when the firm is paying a dividend which having access to external market. The red and pink lines are policies for a firm in environment with low probability of experiencing financing shock ( $\pi = \pi_h$ ). In particular, the pink dotted line is the policy when the firm has access to the external market ( $s = 1$ ). The red solid line is the policy when the firm has no access to external market ( $s = 0$ ) but with existing long-term debt level  $b_L(\pi_h)$ . Again  $b_L(\pi_h)$  is the optimal long-term debt chosen when the firm is paying a dividend which having access to external market. The regions  $[w_L(\tilde{b}_L^-), w_H(\tilde{b}_L^-)]$  and  $[w_L(\hat{b}_L^-), w_H(\hat{b}_L^-)]$  are where the firm will pay dividend if  $s = 1$  and will not pay dividend if  $s = 0$  for the low  $\pi_l$  and the high  $\pi_h$  environments. The parameter values used in the numerical example are listed at the top of figure. I used  $f(k) = k^\alpha$  as the production function and  $A(z)$  is uniform distributed from 0.05 to 0.53.

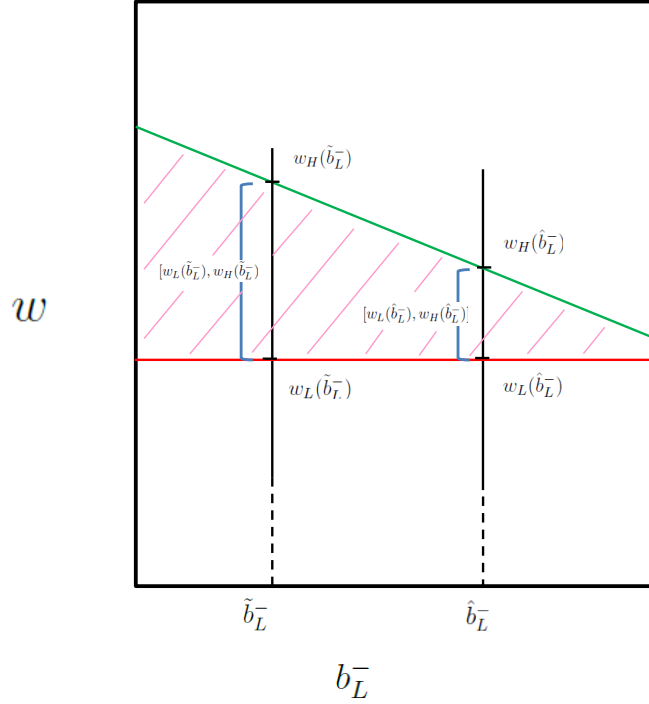


FIGURE 1.3: **Dividends Payout Region.**

The figure shows the stationary distribution of net worth under the ergodic. The y-axis is (current) net worth level and x-axis is existing long-term debt level. The red horizontal line is the dividend paying threshold when the firm has access to external market ( $s = 1$ ). The green downward sloping line is the dividend paying threshold when the firm does not have access to external market ( $s = 0$ ).  $\tilde{b}_L^-$  and  $\hat{b}_L^-$  are two levels of existing long-term debt with  $\tilde{b}_L^- < \hat{b}_L^-$ . Each of the two vertical lines is one locus of optimal next period net worth when the firm currently has access to external market and chooses long-term debt level  $\tilde{b}_L^-$  or  $\hat{b}_L^-$ . The segments  $[w_L(\tilde{b}_L^-), w_H(\tilde{b}_L^-)]$  and  $[w_L(\hat{b}_L^-), w_H(\hat{b}_L^-)]$  are the interval of net worth in which the firm will pay dividend if  $s = 1$  and will not pay dividend if  $s = 0$  for the low  $\pi_l$  and the high  $\pi_h$  environments. The shaded region depicts all such intervals of net worth for all levels of existing long-term debt.

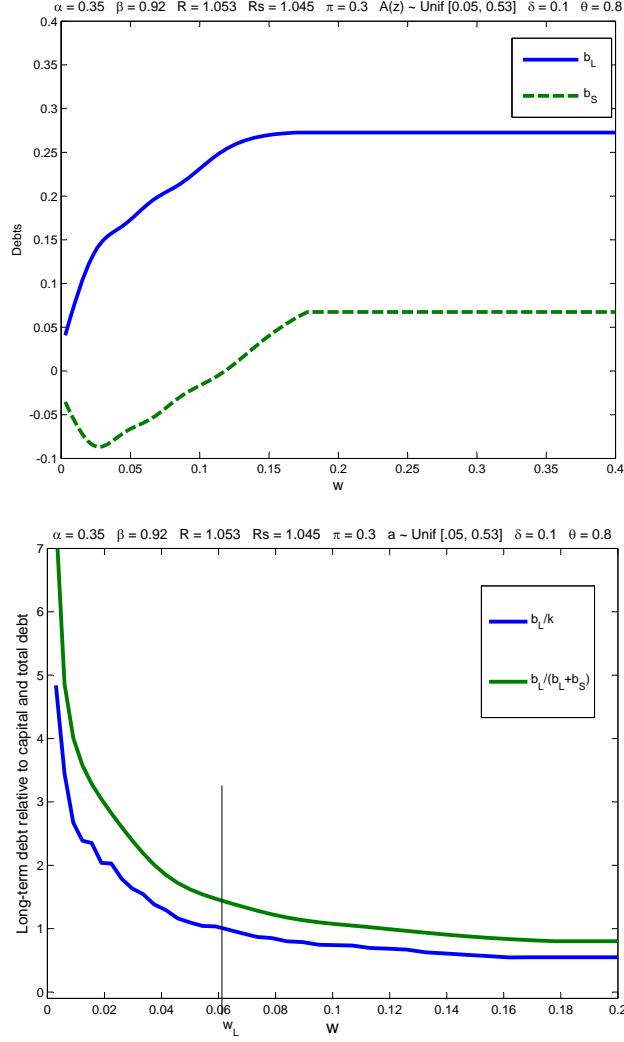


FIGURE 1.4: Long-term v.s. Short-term Debt.

The figure on the top shows the optimal long-term and short-term debt policies as functions of net worth. The y-axis is long-term or short-term debt level and x-axis is net worth. The blue solid line is the optimal long-term debt policy and the green broken line is the optimal short-term debt policy. The figure on the bottom shows the optimal long-term debt to capital ratio and the optimal long-term to total debt ratio as functions of net worth. The y-axis is the ratios and x-axis is net worth. The blue solid line is the optimal long-term debt to capital ratio and the green line is the optimal long-term to total debt ratio.  $w_L$  is the lowest net worth level under the stationary distribution. The parameter values used in the numerical examples are reported in line above the each graph. I used  $f(k) = k^\alpha$  as the production function and  $A(z)$  is uniform distributed from 0.05 to 0.53.

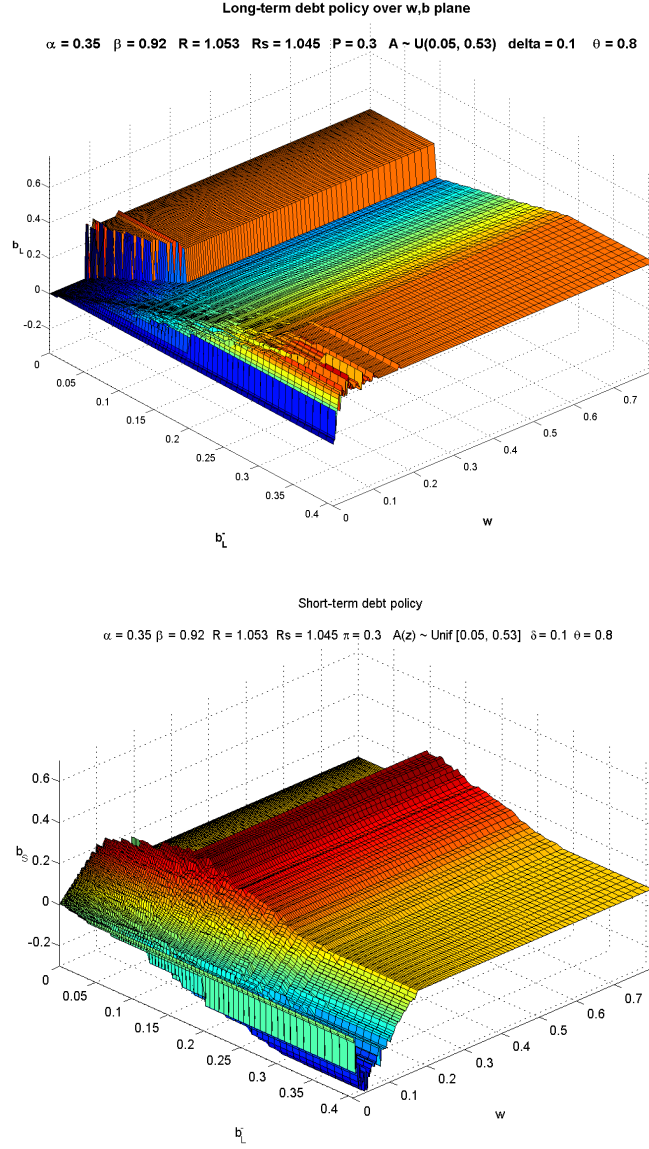


FIGURE 1.5: Debt Policies with Fixed Costs.

The graph on the top shows the optimal long-term debt policy  $b_L$  as a function of net worth  $w$  and existing long-term debt  $b_L^-$ . The x-axis is net worth. The y-axis is existing long-term debt. The z-axis is optimal long-term (short-term) debt choice. The graph on the bottom shows the optimal short-term debt policy  $b_S$  as a function of net worth  $w$  and existing long-term debt  $b_L^-$ . The parameter value used in the numerical examples are reported in line above the each graph. I used  $f(k) = k^\alpha$  as the production function and  $A(z)$  is uniform distributed from 0.05 to 0.53.

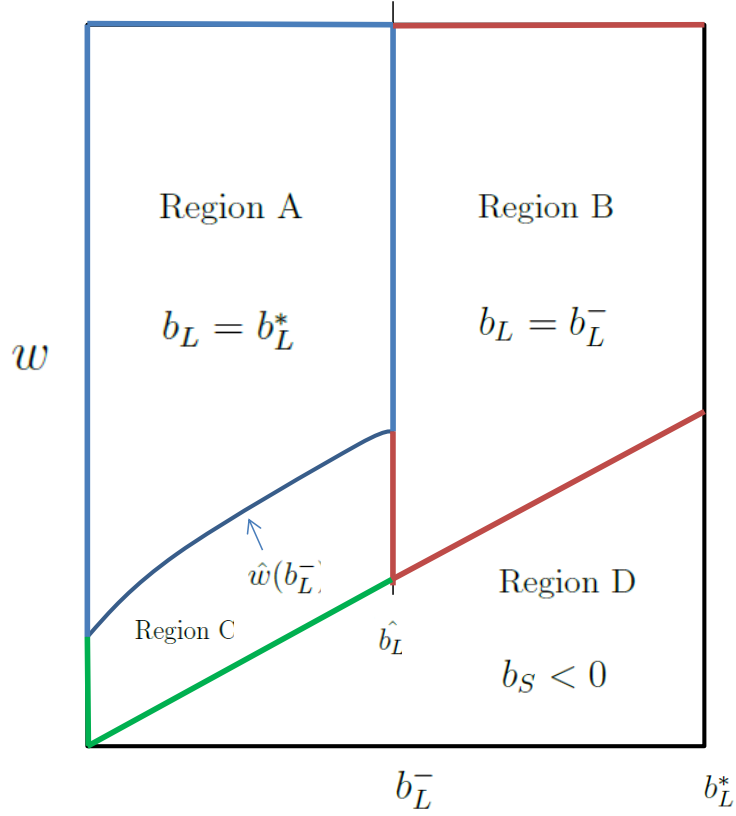
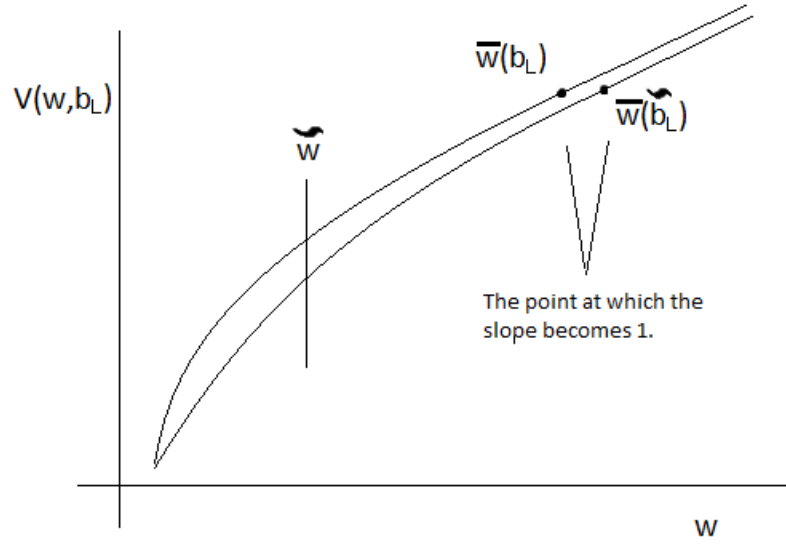


FIGURE 1.6: Contour of Debt Policies with Fixed Costs.

The graph summarizes the regions in which the firms' optimal long-term and short-term decisions show different patterns. The y-axis is net worth and the x-axis is existing long-term debt. Region A is where the firm has sufficiently high net worth and sufficiently low existing long-term debt. Region B is where the firm has high net worth and high existing long-term debt. Region C is where the firm does not have sufficient net worth but with low existing long-term debt. Finally Region D is where the firm has too much long-term debt relative to its net worth.  $\hat{w}(b_L^-)$  is the threshold above which the firm starts to pay the fixed cost and raise long-term debt. The half green and half red upward sloping line is the locus of points where  $b_L^- = \frac{1-p_L}{p_L}w$ .



$u(b_L)$  and  $u(\tilde{b}_L)$  are the slopes, the result have to hold for any arbitrary concave funtions satisfying the conditioins in the proof.

FIGURE 1.7: Value Functions for Two Long-term Debt Levels.

This figure is an illustration for the proof of Proposition 4. The top curve is the value function for the lower long-term debt level  $b_L^-$ . The bottom curve is the value function for the higher long-term debt level  $\tilde{b}_L^-$ .  $\bar{w}(b_L^-)$  and  $\bar{w}(\tilde{b}_L^-)$  are the thresholds at which the firm starts to pay dividends for  $b_L^-$  and  $\tilde{b}_L^-$  respectively.  $\tilde{w}$  is the level of net worth at which the slope of the bottom curve becomes higher than the slope of the top curve.

Table 1.1: **Summary Statistics.**

This table presents the mean, standard deviation, minimum value, median, and maximum value for the variables used in the debt maturity regression (Equation 1.14). The total number of observations and the number of observations identified as Post-reduction are also included at the bottom.

	mean	sd	min	p50	max
Maturity3	0.42	0.34	0.00	0.41	1.00
Log(AT)	5.33	2.07	1.61	5.10	12.42
Market NW	0.61	0.23	0.02	0.64	0.98
Marginal-Q	1.75	1.19	0.22	1.35	6.91
Oper. income	0.10	0.19	-1.11	0.13	0.56
Leverage	0.25	0.19	0.00	0.23	1.13
Asset maturity	8.23	6.18	0.30	6.53	28.22
CF Vol	0.03	0.05	0.00	0.02	0.32
Term spread	1.21	1.14	-0.36	0.83	3.06
Default prob	0.05	0.15	0.00	0.00	0.68
Whited Wu	-0.27	0.11	-0.54	-0.26	-0.02
Observations	20282		Post.	8431	



Table 1.2: **Financial Constraints and Debt Maturity.**

This table presents the estimates from the debt maturity regression (Equation 1.14). All variables other than Postreduction are control variables. The coefficient on Postreduction is the difference in differences. \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%. t-values are given in the parentheses below the estimates.

	(1) Maturity3 b/t	(2) Maturity3 b/t
Postreduction	-0.033*** (-4.0)	-0.027*** (-3.9)
Log(AT)	0.060*** (7.6)	0.039*** (8.6)
Marginal-Q	-0.003 (-0.5)	-0.014*** (-3.0)
Leverage	0.494*** (13.2)	0.596*** (21.8)
Market NW	0.137*** (4.1)	0.046 (1.5)
Oper. income	0.073*** (2.6)	0.104*** (4.8)
Default prob	-0.167*** (-7.6)	-0.316*** (-14.6)
Asset maturity	-0.000 (-0.0)	0.002*** (2.7)
Term spread	-0.006*** (-3.2)	-0.008*** (-4.0)
CF Vol	-0.143 (-1.6)	-0.340*** (-4.7)
Fixed Effects	Firm	SIC
Cluster	Firm	Firm
Observations	20282	20282
Adjusted $R^2$	0.523	0.249

Table 1.3: **Investment Regression Full Panel.**

This table presents the estimates from the investment-debt sensitivity regression (Equation 1.15). The regression is conducted over the observations from the full sample from 1967 to 2001. The first column shows the estimates for the basic regression. The second column shows the estimates for the regression with additional control variables. \* significant at 10%; \*\* significant at 5%; \* \* \* significant at 1%. Standard errors are given in the parentheses below the estimates.

	(Basic)	(w/ Controls)
	$\frac{I_{it}}{K_{it-1}}$	$\frac{I_{it}}{K_{it-1}}$
	b/se	b/se
$q_{it-1}$	0.079*** (0.002)	0.072*** (0.002)
$\frac{CF_{it}}{K_{it-1}}$	0.036*** (0.002)	0.020*** (0.003)
$\frac{DR_{it}}{K_{it-1}}$	-0.264*** (0.018)	-0.252*** (0.018)
ROA		0.153*** (0.019)
Leverage		0.034*** (0.011)
Whited Wu		-0.033 (0.042)
CF Vol		-0.029 (0.021)
Log(MV)		0.015*** (0.002)
Firm Fixed Effects	Yes	Yes
Year Fixed Effects	Yes	Yes
Cluster	Firm	Firm
Observations	73006	72461
Adjusted $R^2$	0.357	0.361

Table 1.4: **Investment Regression Whited Wu Groups.**

This table presents the estimates from the investment-debt sensitivity regression (Equation 1.15). The regression is conducted over five sub-samples from 1967 to 2001. The sub-samples are constructed based on the firms' Whited Wu indices. For each year I sort all firms based on their Whited Wu indices. From the left to right, Group (1) consists of firms with Whited Wu indices in the lowest quintile in each year. Group (5) consists of firms with Whited Wu indices in the highest quintile in each year. Higher Whited Wu index means the firm is more constrained. Hence, from left to right, Groups (1) to (5) are increasing in the degree for financial constraints. Group (1) is the least constrained and Group (5) is the most constrained. \* significant at 10%; \*\* significant at 5%; \* \* \* significant at 1%. Standard errors are given in the parentheses below the estimates.

	(1)	(2)	(3)	(4)	(5)
	$\frac{I_{it}}{K_{it-1}}$	$\frac{I_{it}}{K_{it-1}}$	$\frac{I_{it}}{K_{it-1}}$	$\frac{I_{it}}{K_{it-1}}$	$\frac{I_{it}}{K_{it-1}}$
	b/se	b/se	b/se	b/se	b/se
$q_{it-1}$	0.038*** (0.004)	0.056*** (0.005)	0.084*** (0.006)	0.102*** (0.006)	0.077*** (0.006)
$\frac{CF_{it}}{K_{it-1}}$	0.107*** (0.012)	0.085*** (0.013)	0.057*** (0.009)	0.041*** (0.005)	0.011*** (0.004)
$\frac{DR_{it}}{K_{it-1}}$	-0.071* (0.037)	-0.147*** (0.031)	-0.234*** (0.042)	-0.265*** (0.047)	-0.352*** (0.038)
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Cluster	Firm	Firm	Firm	Firm	Firm
Observations	14585	14611	14604	14611	14595
Adjusted $R^2$	0.506	0.483	0.468	0.380	0.278

Table 1.5: **Investment Regression Size Groups.**

This table presents the estimates from the investment-debt sensitivity regression (Equation 1.15). The regression is conducted over five sub-samples from 1967 to 2001. The sub-samples are constructed based on the firms' size (total asset value). For each year I sort all firms based on their size. From the left to right, Group (1) consists of firms with size in the lowest quintile in each year. Group (5) consists of firms with size in the highest quintile in each year. Bigger firms are generally less constrained. Hence, from left to right, Groups (1) to (5) are decreasing in the degree for financial constraints. Group (5) is the least constrained and Group (1) is the most constrained. \* significant at 10%; \*\* significant at 5%; \* \* \* significant at 1%. Standard errors are given in the parentheses below the estimates.

	(1) $\frac{I_{it}}{K_{it-1}}$ b/se	(2) $\frac{I_{it}}{K_{it-1}}$ b/se	(3) $\frac{I_{it}}{K_{it-1}}$ b/se	(4) $\frac{I_{it}}{K_{it-1}}$ b/se	(5) $\frac{I_{it}}{K_{it-1}}$ b/se
$q_{it-1}$	0.076*** (0.005)	0.099*** (0.006)	0.094*** (0.006)	0.069*** (0.005)	0.044*** (0.004)
$\frac{CF_{it}}{K_{it-1}}$	0.023*** (0.004)	0.035*** (0.005)	0.036*** (0.006)	0.055*** (0.008)	0.074*** (0.010)
$\frac{DR_{it}}{K_{it-1}}$	-0.412*** (0.044)	-0.313*** (0.039)	-0.293*** (0.041)	-0.203*** (0.029)	-0.087** (0.034)
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Cluster	Firm	Firm	Firm	Firm	Firm
Observations	14585	14610	14606	14610	14595
Adjusted $R^2$	0.302	0.398	0.450	0.462	0.475

Table 1.6: **Investment Regression Dividend Groups.**

This table presents the estimates from the investment-debt sensitivity regression (Equation 1.15). The regression is conducted over five sub-samples from 1967 to 2001. The sub-samples are constructed based on the firms' dividends to EBITDA ratios. Group (1) consists of firms with negative EBITDA. Group (2) consists of firms with dividends to EBITDA ratios equal to zero. Group (3) consists of firms with dividends to EBITDA ratios between 0 and 0.13. Group (4) consists of firms with dividends to EBITDA ratios greater than 0.13. Generally, firms that pay more dividends are considered to be less constrained. Also, firms that have negative EBITDA are considered to be very constrained. Hence, from left to right, Groups (1) to (4) are decreasing in the degree for financial constraints. Group (4) is the least constrained and Group (1) is the most constrained. \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%. Standard errors are given in the parentheses below the estimates.

	(1) $\frac{I_{it}}{K_{it-1}}$ b/se	(2) $\frac{I_{it}}{K_{it-1}}$ b/se	(3) $\frac{I_{it}}{K_{it-1}}$ b/se	(4) $\frac{I_{it}}{K_{it-1}}$ b/se
$q_{it-1}$	0.071*** (0.006)	0.063*** (0.004)	0.053*** (0.005)	0.029*** (0.004)
$\frac{CF_{it}}{K_{it-1}}$	-0.029*** (0.004)	0.172*** (0.007)	0.164*** (0.015)	0.102*** (0.013)
$\frac{DR_{it}}{K_{it-1}}$	-0.301*** (0.038)	-0.296*** (0.039)	-0.127*** (0.031)	-0.099*** (0.033)
Constant	0.183*** (0.032)	0.256*** (0.039)	0.197*** (0.017)	0.188*** (0.010)
Firm Fixed Effects	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Cluster	Firm	Firm	Firm	Firm
Observations	13727	18905	21411	18960
Adjusted $R^2$	0.320	0.462	0.412	0.379

Table 1.7: **Investment Regression Reduction in Tariff.**

This table presents the estimates from the investment-debt sensitivity regression (Equation 1.16). The first column shows the estimates for the basic regression. The second column shows the estimates for the regression with additional control variables. \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%. t-values are given in the parentheses below the estimates.

	(Basic)	(w/ Controls)
	$\frac{I_{i,j,t}}{K_{i,j,t-1}}$	$\frac{I_{i,j,t}}{K_{i,j,t-1}}$
	b/t	b/t
$\frac{DR_{i,j,t}}{K_{i,j,t-1}}$	-0.345***	-0.299***
	(-8.6)	(-6.4)
$Post_{j,t}$	-0.007	-0.048***
	(-0.9)	(-7.5)
$(\frac{DR_{i,j,t}}{K_{i,j,t-1}} \times Post_{j,t})$	-0.073**	-0.060*
	(-2.3)	(-1.9)
$q_{i,j,t-1}$	0.080***	0.071***
	(19.6)	(15.0)
$\frac{CF_{i,j,t}}{K_{i,j,t-1}}$	0.025***	0.001
	(5.7)	(0.2)
Market NW		0.107***
		(4.2)
Tangibility		0.196***
		(4.5)
Whited Wu		-0.299***
		(-3.6)
Term spread		-0.011***
		(-7.7)
Year Fixed Effects	Yes	No
Firm Fixed Effects	Yes	Yes
Cluster	Firm	Firm
Adjusted $R^2$	0.430	0.427

Table 1.8: **Definition of Variables.**

This table presents the definitions of main variables. Compustat data are from the annual and quarterly Compustat database. Bond yields are from the Federal Reserve Bank - St. Louis. The construction of “Postreduction” is stated in section 1.7.1.

Variables	Definition
Maturity3	Fraction of debt due more than 3 years. That is long-term debt due more than 3 years divided by (Long-term debt + debt in current liabilities)
Log(AT)	The natural log of total assets.
Marginal-Q, $q_{i,t}$	The market value of the firm divided by the book value of the firm. The book value of firm is total assets. Market value of firm is total assets + market value of share outstanding - book value of equity.
Market NW	The market value of net worth divided by the market value of the firm. The market value of net worth is the market value of the firm - total liabilities
Oper. income	Operating income before depreciation divided by lagged total assets
Leverage	(Long-term debt + debt in current liabilities) divided by total assets
Asset maturity	(Gross property, plant, and equipment (PP&E)/total assets) $\times$ (gross PP&E/depreciation expense) + (current assets/total assets) $\times$ (current assets/cost of goods sold)
CF Vol	Cash flow volatility calculated as the ratio of the standard deviation of the past eight earnings changes to the average total assets over the past eight quarters
Term spread	The difference between 10-year Treasury yield and the 3-month T-bill yield
Default prob	Firm default probability estimated by the “naive” approach as in Bharath and Shumway (2008)
Whited Wu	$-.091 \times \text{cash flow/total assets} - .062 \times \text{indicator for paying dividend} + .021 \times \text{liabilities/total assets} - .044 \times \text{total assets}$ $+ .102 \times \text{industry sales growth} - .035 \times \text{firm sales growth}$
Log(MV)	The natural log of market value of net worth. Market value of net worth is the market value of firm - total liabilities
Tangibility	Net property, plant, and equipment divided by total asset
Investment Rate, $\frac{I_{i,t}}{K_{i,t-1}}$	Capital expenditure divided by lagged net property, plant, and equipment
Cash Flow, $\frac{CF_{i,t}}{K_{i,t-1}}$	(Income before extraordinary items + depreciation) divided by lagged net property, plant, and equipment
Debt Repayment, $\frac{DR_{i,t}}{K_{i,t-1}}$	Debt in current liabilities divided by lagged net property, plant, and equipment

## Resource Allocation and Debt Maturity

### 2.1 Introduction

In the previous chapter, I studied a single firm's debt maturity decision when facing future rollover risks. Now I investigate the aggregate implications. In recent years, many study the effects of capital misallocation on aggregate productivity (Moll (2014), Midrigan and Xu (2014), etc). When financial frictions prevent the most productive agents in the economy to raise enough capital, some capital remains in the hands of less productive agents. Hence, the aggregate productivity is lower than the first best level. When more capital is misallocated, the aggregate productivity drops more. The use of short-term financing leads to more rollover problems. When old debt is due and new external financing is difficult, productive firms have to downsize and are unable to deploy enough capital. Therefore, capital is misallocated and aggregate productivity falls. Naturally, it seems that more use of long-term debt in the economy can reduce the rollover risks and alleviate capital misallocation. However, in reality long-term debt may also cause capital misallocation. For example, suppose a firm issues a long-term debt to raise capital for the current production, but it becomes unproductive in all future periods; unless the firm can reinvest in the market at the same interest



rate or unless the debt is callable, the firm will have to produce at a low rate and keep paying the interest on the debt. On the other hand, suppose an entity lends resources with a long-term contract, then it comes across a productive project; unless the entity can ask for early total repayment on the loan or it can borrow from the external market, the entity will not be able to deploy enough capital for the project. Therefore, when there are frictions in both borrowing and lending, whether long-term debt can help reduce misallocation depends on whether productivity is predictable. To what degree long-term debt is helpful depends on how predictable future productivity is. The use of long-term and short-term debt on the aggregate level should be related to the predictability of future productivity. In this chapter, I explore the connections between idiosyncratic productivity and aggregate debt maturity. I show why agents in the economy sometimes choose to allocate resources more with long-term debt and other times choose to allocate more with short-term debt. I explain how changes in the predictability of total factor productivity (TFP) drive variations in aggregate debt maturity. I also examine the effects of debt maturity on misallocation and aggregate productivity.

Conventional wisdom would say that when cash flows become more volatile, firms will want to reduce the amount of debt repayments in the short run by borrowing with long-term debt instead of short-term debt. However, looking at the problem only from the firm's point of view is misleading. In equilibrium, the interest rate or the price of long-term debt is determined taking into account of each firm's incentive to borrow long-term debt in order to endure times with high uncertainty. Therefore, on the aggregate level debt maturity may not increase with rising uncertainty. In fact, empirical evidence suggests that on the aggregate level, corporate debt maturity shortens during recessions or during financial crisis. Usually, both real and financial bad times are associated with high uncertainty. Hence, it seems that aggregate debt maturity drops as uncertainty rises. With a general equilibrium setup, my model

points out that the decrease in debt maturity is a direct response to the increase in uncertainty in the idiosyncratic productivity. More specifically, the entrepreneurs will use more short-term debt when productivity becomes less predictable.

The productivity process indeed changes from period to period. Some supports are shown in the study of counter-cyclical uncertainty in production induced by demand conditions (Bloom et al. (2007) and Bloom (2009)). Bloom and his co-authors document that idiosyncratic productivity becomes more volatile following disasters and in bad times. Also, Eisfeldt and Rampini (2006) point out that the dispersion in productivity of firms is counter-cyclical. The increase in the volatility of TFP in bad times could lead to a counter-cyclical dispersion. Moreover, as I will show in this chapter, productivity is more persistent during expansions and less persistent during recessions. Both facts suggest that future productivity is harder to predict during economic downturn. Taking those facts as given, I build a dynamic general equilibrium model in which entrepreneurs can at times allocate capital resources among one another subject to collateral constraints. Other times entrepreneurs may fail to find counter-parties, hence they may not be able to borrow or lend in the current period. With persistent productivity and potential future exclusion from external capital market, entrepreneurs have incentives to arrange long-term transfer of resources while they are able to borrow and lend. The degree to which the entrepreneurs will conduct long-term transfer depends on how well they can predict their future productivity. When productivity can be more precisely predicted, the entrepreneurs will transfer more through long-term borrowing and lending. In the extreme case, suppose the productivity is permanent and constant, the entrepreneurs will always use long-term debt and never use any short-term debt. However, if the productivity is serially uncorrelated, then the entrepreneurs may not use long-term debt at all since the resource is likely to be stuck in the wrong hand in the future. Hence, it is better for the entrepreneurs to reallocate every period with short-term debt. My

model provides guidelines on how the predictability of the idiosyncratic TFP relates to aggregate debt maturity. In particular, my model explains that counter-cyclical predictability of TFP leads to pro-cyclical aggregate debt maturity.

The pro-cyclical aggregate maturity of corporate debt has been well documented recently. Main and Santos (2011) find that the value weighted average debt maturity of all corporate debt in the economy is longer in good times and shorter in bad times. Although their original paper considers financial good times and bad times, those periods mostly coincide with expansions and recessions. They conjecture that firms take more long-term debt as a precaution against difficult times in future. In indeed, they discovered that the firms with strong balance sheets lengthen their current debt and expand unused lines of credit during expansions. On the other hand, firms take on more short-term debt and draw down their lines of credit during recessions. Chen, Xu, and Yan (2013) also document the same pro-cyclical pattern. In their model, long-term debt reduces the firm's default risks. However, the long-term debt makes the investors more exposed to liquidity shocks. During an expansion, firms issue long-term debt so that they may have less debt maturing in the near future. When recession comes, risk premium rises dramatically, so long-term debt is too expensive to obtain. Hence firms switch to short-term debt. I take a look at the issue from a aggregate point of view. Instead of relying on the premises of counter-cyclical risk premium, I propose that the pro-cyclical pattern of aggregate debt maturity is driven by the nature of the productivity process.

Existing literature commonly blames short-term financing for exacerbating the problems in bad times. My model suggests that the more use of short-term debt can also be a natural responds to the increasing uncertainty in productivity. Because the productivity is hard to predict in bad times, more reshuffle of resources is needed each period. Hence, more short-term debt is used. Indeed, my model shows that with low persistence in TFP, the benefit of long-term debt is small and the loss

in aggregate productivity is marginal. Nevertheless, the aggregate productivity of entrepreneurs that are currently excluded from the external market is much lower in the recessions than in the expansions. Hence there is a significant loss in aggregate productivity from capital misallocation during recessions. Moreover, my model quantifies the importance of long-term debt in alleviating capital misallocation and preserving aggregate productivity when entrepreneurs are excluded from the external capital market. Finally, my model illustrates that a financial crisis changes aggregate productivity more dramatically in expansions than in recessions.

The rest of the Chapter proceeds as follows. Section 2.2 outlines the economic environment. Section 2.3 presents the individual's model and solutions. Section 2.4 describes the equilibrium and discusses the dynamics of the economic system. Section 2.5 investigates the relation of debt maturity and aggregate TFP. Finally, section 2.6 concludes the chapter.

## 2.2 Environment

In a frictionless economy, resources are always allocated to the most productivity agents, therefore each realization of idiosyncratic TFP does not affect the overall production on the aggregate level. Moreover, on the micro level, each agent does not have to worry about future resource allocation, he only considers his borrowing or lending decision on the spot for each period repetitively. However, When facing financial frictions, entrepreneurs are sometimes unable to adjust their financial structure. Hence they will tend to adjust more when they have the chance to do so. In particular, they will use long-term debt to arrange long-term transfers in case they will not be able to transfer in the future. When the volatility of the TFP shocks is high or when the TFP process is less persistence, current TFP is less informative about future TFP. Hence entrepreneurs would like to adjust through short-term contacts and adjust more each period while taking more risk of stuck with suboptimal financial structure. To study

this intuition more rigorously, I build a general equilibrium model with the following setup.

There is a continuum of entrepreneurs each identified by his or her own net worth  $a$ , existing long-term debt  $b^-$ , and TFP  $z$ . The entrepreneurs can borrow and lend resources to one other, produce with capital and labor, and consume the output. The entrepreneurs have log utility and discount future utility at rate  $\beta$ . Due to some exogenous financial frictions, the entrepreneur may not be able to borrow or lend with probability  $p$ . For future convenience, I say an entrepreneur is “in-the-market” if he can borrow and lend and denote such a state with  $s = 1$ . I say an entrepreneur is “out-of-market” if he cannot borrow or lend and denote the state with  $s = 0$ . The exact micro-foundations for this frictions will be future works by itself. One could think of it as a search friction where with probability  $p$  the entrepreneur does not find a counter-party. The main results do not depend on this specific form of formulation. In fact, I could use exogenous financing cost function that assign different costs of borrowing and lending for a continuum of states.

In order to focus on the long-term debt’s role as a prearrangement vehicle, I stay away from the entrepreneurs’ debt manipulating practice arise from the difference in interest rates. To do so, I put restriction on the form of long-term debt payoff. I require that the long-term debt pays the same as the short-term debt does when the entrepreneur is in the market and pays a different rate if the entrepreneur is “out-of-market”. So that the difference in interest rate occurs only in “out-of-market” state where the entrepreneurs cannot adjust their capital structure.

The timeline is shown in Figure 2.1. Each day, the entrepreneur wakes up with his net worth,  $a$ , and money borrowed last period,  $b^-$ . Also, he observes his productivity  $z$  and learns whether he is “in-the-market”. If he is “in-the-market”, in the morning, before production he can borrow new short-term and long-term debt subject to a collateral constraint. Then, he uses the money (own and borrowed) to buy capital

and hire labor to carry out the production. After the production, in the afternoon he repays the interest and debt. More specifically, he pays the current interest on the new long-term debt,  $rb$ , the interest and principle on old long-term debt,  $Rb_{-1}$ , and new short-term debt,  $Rd$ . Finally, in the evening, he decides how much net worth to keep for tomorrow,  $a'$ , and consumes the rest.

Let  $X$  summarizes all the macro state variables including the entire distributions of both “in-the-market” and “out-of-market” entrepreneurs. Formally, an entrepreneur faces the following problem.

### 2.3 Individual Problem

$$V(a, b^-, z, s; X) = \max_{d, b, k, l, c, a'} \ln(c) + \beta E_{z'} \{pV(a', b, z', 0; X') + (1-p)V(a', b, z', 1; X')\} \quad (2.1)$$

$$k \leq a + b + b^- + d, \quad (2.2)$$

$$0 \leq a' + b, \quad (2.3)$$

$$0 \leq k, \quad 0 \leq l. \quad (2.4)$$

for  $s = 1$  :

$$b + b^- + d \leq (\lambda - 1)a, \quad 1 \leq \lambda, \quad (2.5)$$

$$a' \leq (zk)^\alpha l^{1-\alpha} - wl + (1 - \delta)k - R(X)d - R(X)b^- - R(X)b - c, \quad (2.6)$$

for  $s = 0$  :

$$a' \leq (zk)^\alpha l^{1-\alpha} - wl + (1 - \delta)k - R(X)d - R_L(X^-)b^- - R(X)b - c, \quad (2.7)$$

$$0 \leq b \leq 0, \quad 0 \leq d \leq 0. \quad (2.8)$$

As usual, the entrepreneur maximizes the sum of current utility and discounted expected future continuation value. The expectation is over both the next period TFP states,  $z$ , and market conditions (in or out of market),  $s$ . Constraint (2.2) says that

the total capital used in production must be financed out of the entrepreneur's own net worth and external borrowing. Constraint (2.3) makes sure that the entrepreneur can only lend out as much as his own net worth at the end of the period. Collateral constraint (2.5) states that the entrepreneur can only borrow up to a fraction  $(\lambda - 1)$  of its net worth. Hence, one can think of  $\lambda$  as the maximal leverage for the entrepreneur. The budget constraints (2.6) and (2.7) govern the laws of motion for the entrepreneur's net worth. The entrepreneur's end of period net worth is his profit from production net of all liabilities and consumption. Although the principle of the long-term debt is not due at the end of the current period, I still exclude it from the net worth calculation. Since end period net worth determines the maximal leverage for next period, this definition of net worth prevent levering on borrowed money. Also, the difference between constraint (2.6) and (2.7) is that when the entrepreneur is "out-of-market", he pays a different rate  $R_L$  on the long-term debt. This rate  $R_L$  will be pinned down in equilibrium and it will balance the demand and supply of long-term debt. Finally, if the entrepreneur is "out-of-market", then he is unable to borrow or lend through either short or long term debt as stated by the set of constraints (2.8).

Next, I characterize the solution of the model. By setup, the capital, labor, and short-term debt decisions do not involve intertemporal trade-offs. Also, while in the market, the entrepreneur can always adjust the total debt level with short term debt subject to the linear collateral constraint and non-negativity constraint on capital. Hence, I can separate the problems in the following way. First, the entrepreneur chooses capital, labor, and total leverage to maximize his current wealth. Then he decides on consumption, end period net worth, and long-term debt. The short-term debt will be the difference between the total debt and the long-term debt (both old and newly issued).

### 2.3.1 Capital, labor, and total debt

The solution to the entrepreneur's within period profit maximization problem is summarize by the following lemma:

**Lemma 13.**

$$k(a, b^-, z, 1) = \begin{cases} \lambda a, & \text{if } z \geq \bar{z} \\ 0, & \text{if } z < \bar{z} \end{cases}$$

$$\text{where } R(X) = \bar{z}(X)\pi(X) + 1 - \delta,$$

$$\pi = \alpha \left( \frac{1 - \alpha}{W(X)} \right)^{(1-\alpha)/\alpha}.$$

$$k(a, b^-, z, 0) = a + b^-, \quad l(a, b^-, z, s) = \left( \frac{1 - \alpha}{W(X)} \right)^{\frac{1}{\alpha}} z k$$

First, the optimal labor decision is always linear in productivity and optimal capital level. When entrepreneurs are in the market, they lever up and produce at their maximal scale allowed by the collateral constraint if they productivities are higher than a cut-off level,  $\bar{z}$ . Otherwise, they lend out all resources. When entrepreneurs are out of market, they produce with all their resources regardless the productivity level since they are living in autarky. Hence, the entrepreneur's profits from production,  $\Pi(a, b^-, z, s) = (zk)^\alpha l^{1-\alpha} - Wl + (1 - \delta)k$ , for state  $s = 0$  and state  $s = 1$  can be reduced to  $\Pi(a, b^-, z, 1) = \max\{z\pi + 1 - \delta, 0\}k$  and  $\Pi(a, b^-, z, 0) = [z\pi + 1 - \delta]k$  respectively. Those are standard results due to the constant return to scale technology as in Moll (2014).

**Lemma 14.** *Existing long-term debt  $b^-$  only affects short-term debt choice when entrepreneurs are currently in the market.*

When an entrepreneur is “in-the-market”, he can use both long-term and short-term debt subject to a collateral constraint on the total amount of borrowing. The maximum amount of resource he can borrow or lend is pinned down by his net worth.



Plus, the long-term and short-term interest rates are the same. Hence, the old long-term debt outstanding will have no effects. In fact, any old long-term debt outstanding can be counted towards short-term debt.

Therefore, the problem can be reduced to the following consumption-saving and portfolio choice problem.

### 2.3.2 Consumption, net worth and long-term debt

With optimal profit and leverage, I can rewrite the remaining problem as a simple consumption, saving, and portfolio choice problem:

$$V(a, b^-, z, s) = \max_{a', b} \ln(c) + \beta E_{z'} \{pV(a', b, z', 0) + (1-p)V(a', b, z', 1)\}$$

$$a' = [\lambda \max\{z\pi - r - \delta, 0\} + R]a - c, \quad 0 \leq a' + b, \quad \text{for } s = 1$$

$$a' = (z\pi + 1 - \delta)(a + b^-) - R_L b^- - c \quad b = 0, \quad \text{for } s = 0$$

where  $\pi = \alpha(\frac{1-\alpha}{W})^{(1-\alpha)/\alpha}$ .

Now let's denote the returns on saving for states  $s = 1, 2$  as  $A(z, 0) = z\pi + 1 - \delta$  and  $A(z, 1) = \lambda \max\{z\pi - r - \delta, 0\} + R(X)$  respectively. Basically,  $A(z, s)$  is the return on net worth (or internal funds) for state  $(z, s)$ . Let  $\eta_{a'+b}$  be the multiplier on constraint (2.3). I obtain the following optimality conditions.

Consumption Saving: (proportion to be consumed) for  $s = 0, 1$

$$\frac{1}{c(a, b^-, z, s)} = \beta \int_0^\infty \left( \frac{pA(z', 0)}{c(a', b, z', 0)} + \frac{(1-p)A(z', 1)}{c(a', b, z', 1)} \right) \psi(z'|z) dz' + \eta_{a'+b},$$

Portfolio Choice: (proportion in long-term debt) for  $s = 1$

$$\int_0^\infty \frac{A(z', 0) - R_L}{c(a', b, z', 0)} \psi(z'|z) dz' + \eta_{a'+b} = 0,$$

Since the entrepreneur can always use short-term debt to make up the gap when he is “in-the-market”, The long-term debt decision does not directly affect the entrepreneur's saving decision through the budget constraint when  $s = 1$ . In general, if

he thinks that he will likely be very productive next period, he will borrow through long-term debt to reduce its debt repayments at the end of the period. Hence, he won't have to downsize as much even if he is "out-of-market" next period. On the other hand, if he thinks that he will likely be unproductive next period, he will lend in long-term debt and receive the riskless return of  $R_L$  tomorrow.

With log utility, the saving decision is separated from the long-term debt in the sense that the consumption and saving decisions are not affected by long-term debt choice. When constraint (2.3) is binding, the entrepreneur lends out all he has, hence  $b = -a'$ .

Because of log utility and constant return to scale production, the decision rules are linear in wealth. In our model, since total borrowing is constrained by net worth, wealth is simply the product of net worth,  $a$ , and return on net worth,  $A(z, s)$ . Furthermore, the proportional decisions on consumption, saving, and long-term debt are only functions of current productivity level,  $z$ , and independent of the size or wealth of the entrepreneur. Hence, I am able to characterize the affect of current productivity on entrepreneurs' decisions of what fraction of wealth they will consume, save, and borrow/lend through long-term debt.

As long as  $z\pi + 1 - \delta > 0$ , our result does not depend on the domain of productivity shocks. For convenience I let  $z \in [0, \infty)$ . Assuming  $R_L \geq E(z'|z=0)\pi + 1 - \delta$ , which is always the case in equilibrium, I have the following solution.

**Proposition 15.** *The policy function can be characterized as:*

$$c = (1 - \beta)A(z, 1)a, \quad b = \phi_b(z)A(z, 1)a, \quad a' = \beta A(z, 1)a, \quad \text{for } s = 1$$

$$c = (1 - \beta)\{A(z, 0)(a + b^-) - R_L b^-\}, \quad b = 0, \quad a' = \beta\{A(z, 0)(a + b^-) - R_L b^-\}, \quad \text{for } s = 0$$

$$\phi_b(z) \text{ is defined by } \begin{cases} \int_0^\infty \frac{R_L \psi(z'|z)}{(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b} dz' = \frac{1}{\beta}, & \text{if } z > \underline{z} \\ \phi_b(z) = -\beta, & \text{if } z \leq \underline{z} \end{cases}$$

where  $\underline{z}$  satisfies  $R_L = E(z'|\underline{z})\pi + 1 - \delta$ .

Since I have the closed form solutions, I can easily conduct the following comparative statics.

**Corollary 16.** *Proportionally long-term borrowing  $\phi_b$  is decreasing in  $R_L$ .*

Naturally, when long-term debt becomes more expensive, entrepreneurs borrow less of it.

**Corollary 17.** *Entrepreneurs with higher initial net worth ( $a$ ), consume more, accumulate more net worth, and use more debt. Entrepreneurs with higher current productivity ( $z$ ) consume more and accumulate more net worth.*

As mentions before, all decisions are linear in net worth,  $a$ . Therefore, with the same current productivity, bigger entrepreneurs make decisions on a large scale. The effect of  $z$  on long-term debt,  $b$ , is subtle. Current productivity plays two roles. First, it contributes to current wealth. Second, it provides information about future productivity. To investigate this second effect, I put a commonly accepted structure on the productivity process.

In particular, I assume that the productivity shocks are persistent, so the current productivity level can predict future productivity levels. Hence, more and less productive firms today will make different proportional long-term debt decisions.

**Assumption 4.** *Suppose for any  $z^- < z^+$ ,  $\psi(z'^+)$  first order stochastically dominates  $\psi(z'^-)$ .*

Under the assumption and from long-term debt policy given in proposition 15, I have the following proposition.

**Proposition 18.** *The optimal policy on long-term debt,  $\phi_b(z)$ , is increasing in current productivity,  $z$ . Moreover,  $\phi_b(z) \leq 0$ , for  $z \leq z^b$ , and  $\phi_b(z) \geq 0$ , for  $z \geq z^b$ , where  $z^b$  is defined by  $\int_0^\infty \frac{\psi(z'|z^b)}{z'\pi+1-\delta} dz' = \frac{1}{R_L}$ .*

In corollary 17, we see that more productive entrepreneurs consume more and keep more net worth. However, for long-term debt that is not always the case. Entrepreneurs with current productivity greater than  $z^b$  are borrowers. For them, an increase in  $z$  indeed leads to more borrowing. However, for the lenders, increase in  $z$  leads to a reduction in proportional lending,  $\phi_b$ , but an increase in current wealth,  $A(z, 1)a$ . Therefore, total lending in long-term debt may either rise or fall. Also, Proposition 18 suggests that on an individual level, debt maturity is pro-cyclical. That is the entrepreneurs will borrow more long-term debt when productivity is high as in expansion and borrow less long-term debt when productivity is low as in recession. However, under general equilibrium, the interest rate on the long-term debt may also increase. Hence, it is unclear whether on the aggregate level, debt maturity is pro-cyclical. In the next section, I will investigate what happens on the aggregate level under general equilibrium. But first, let's give a specific form to the TFP process for easier exploration.

Suppose the productivity process is:  $\log(z') = \rho \log(z) - \frac{\sigma^2}{2} + \sigma \varepsilon$ , where  $\varepsilon \sim N(0, 1)$ . I investigate how the persistence of productivity affects the long-term debt policy.

**Proposition 19.**  *$\phi_b$  is increasing in  $\rho$  for  $z > 1$  and decreasing in  $\rho$  for  $z < 1$ .*

The intuition is straight forward. For entrepreneurs with  $z > 1$ , increase in  $\rho$ , raises the condition mean of  $z'$ , hence the future investment opportunity becomes better. Thus, the entrepreneurs borrow more long-term debt. The exact opposite applies for entrepreneurs with  $z < 1$ .

**Proposition 20.**  *$\phi_b$  is decreasing in  $\sigma$ .*

When  $z'$  becomes more volatile, the future investment is riskier, hence entrepreneurs want to engage in more precautionary saving by reducing their long-term debt or lend more through long-term debt.

## 2.4 Equilibrium and Dynamics

Let  $\tilde{G}_t(a, z)$  and  $\tilde{g}(a, z)$  be the cdf and pdf for the measure of entrepreneurs who are currently in the market. Since, existing long-term debt does not matter when the entrepreneurs are in the market. They can be measured simply with net worth  $a$  and productivity  $z$ . Let  $\tilde{H}_t(a, b^-, z)$  and  $\tilde{h}(a, b^-, z)$  be the measure of entrepreneurs who are currently out of the market. The competitive equilibrium is defined by time paths of prices  $R_t, W_t, t \geq 0$  and corresponding quantities, such that (i) taking equilibrium prices as given, each entrepreneur maximizes (2.1) subject to (2.2) to (2.8) , and (ii) the short-term debt, long-term debt, and labor markets clear at each point in time:

$$\begin{aligned} \int_z \int_a k_t(a, z) d\tilde{G}_t(a, z) &= \int_z \int_a a d\tilde{G}_t(a, z), \\ \int_z \int_a b_t(a, z) d\tilde{G}_t(a, z) &= 0, \\ \int_z \int_a l_t(a, z) d\tilde{G}_t(a, z) + \int_z \int_a \int_{b^-} l_t(a, z) d\tilde{H}_t(a, b^-, z) &= L. \end{aligned}$$

The first equation is the market clearing condition for total resources. It states that the total use of resources in production equals the total amount of resources available at the beginning of the period. The second equation is the market clearing condition for long-term debt. It states that overall there is net zero supply of long-term debt since the entrepreneurs simply borrow and lend from one another. The last equation is the market clearing condition for labor. It insures that at the current wage level, all labor units are used by all the entrepreneurs.

The short term debt interest rate is determined by the marginal active (the ones who produce) entrepreneurs in the market. Formally, I have the following relation:  $\int_0^\infty \int_{\bar{z}}^\infty a \tilde{g}(a, z) dz da = \frac{1}{\lambda} \int_0^\infty \int_0^\infty a \tilde{g}(a, z) dz da$ , where  $\frac{1}{\lambda}$  can be viewed as the down payment per unit of investment the active entrepreneurs must provide with their own money. The relation states that the total amount of resources in the hands of active

entrepreneurs is a fraction  $\frac{1}{\lambda}$  of the total resources that are in the market. Later I will discuss in more details about how the short-term interest rate is determined. Next, I discuss how to keep track of the distributions of both in and out of the market entrepreneurs.

Each period, there will be a fraction  $p$  of the entrepreneurs are “out-of-market”. Their decisions are simple and do not affect the interest rates of the debt. Hence, I focus mainly on the distribution and decisions of the entrepreneurs that are in the market. Depending on the realizations of their idiosyncratic productivity shocks, A spectrum of “in-the-market” entrepreneurs can be obtained as follows.

For all the entrepreneurs that are in the market, the entrepreneurs with productivity higher than  $\bar{z}$  will be active and produce. They borrow from the ones with productivity lower than  $\bar{z}$ . The entrepreneurs with productivity higher than  $z^b$  borrow long-term debt. Moreover, the higher the productivity, the more long-term debt is borrowed. On the other hand, The entrepreneurs with productivity lower than  $z^b$  lend in long term. Furthermore, the ones with lower productivity lend more in long term. Finally, there is a cut-off level of productivity  $\underline{z}$  below which all entrepreneurs lend all their net worth out through long-term debt. That is their optimal choice of  $\phi_b$  is  $-\beta$ . The cut-off  $\underline{z}$  is determined by  $R_L = E(z'|\underline{z})\pi + 1 - \delta$ .

Now I derive the distributions of both “in-the-market” and “out-of-market” entrepreneurs. Let  $g_t(a, \phi_b^-, z)$  be the measure of entrepreneurs who are previously “in-the-market” with net worth  $a$ , long-term debt proportion  $\phi_b^-$ , and productivity  $z$ . Let  $h_t(a, 0, z)$  be the measure of entrepreneurs who are previously “out-of-market” with net worth  $a$ , zero long-term debt, and productivity  $z$ . The long-term debt is zero because the entrepreneurs cannot raise long-term debt if they are “out-of-market” in last period.

For entrepreneurs that are previously “in-the-market”, their existing long-term debt is decided in last period based on their productivity in last period  $z_{t-1}$ . I define

the wealth share of “in-the-market” entrepreneur with long-term debt to wealth ratio,  $\phi_b^-$ , and productivity,  $z$ , as:

$$u_t^1(\phi_b^-, z) = \frac{1}{K_t^{in}} \int_0^\infty a g_t(a, \phi_b^-, z) da,$$

where  $g_t(a, \phi_b^-, z)$  is the measure of “in-the-market” entrepreneurs with net worth  $a$ , long-term debt  $\frac{\phi_b^-}{\beta} a$ , and productivity  $z$  and  $K_t^{in} = \int_0^\infty \int_0^\infty \int_{-\beta}^\infty a g_t(a, \phi_b^-, z) d\phi_b^- dz da$  is the total resources that are in the market.

For entrepreneurs that are previously “out-of-market”, the existing long-term debt is zero. Hence, I can track their wealth share with productivity alone. I define the wealth share of “out-of-market” entrepreneur with productivity,  $z$ , as:

$$u_t^0(z) = \frac{1}{K_t^{out}} \int_0^\infty a h_t(a, 0, z) da, \quad (2.9)$$

where  $h_t(a, 0, z)$  is the measure of “out-of-market” entrepreneur with net worth  $a$ , zero long-term debt, and productivity  $z$  and  $K_t^{out} = \int_0^\infty \int_0^\infty a h_t(a, 0, z) dz da$  is the total resource that are out of the market. Similarly, the wealth share of the entrepreneurs in the market with productivity  $z$  should be defined as:

$$\omega_t(z) = \frac{1}{K_t^{in}} \int_{-\beta}^\infty \int_0^\infty a g_t(a, \phi_b^-, z) da d\phi_b^- = \frac{1}{K_t^{in}} \int_0^\infty \int_0^\infty a g_t(a, \phi_b^-(z_{t-1}), z) da dz_{t-1}, \quad (2.10)$$

where  $K_t^{in} = \int_0^\infty \int_0^\infty \int_{-\beta}^\infty a g_t(a, \phi_b^-, z) d\phi_b^- dz da$  is the total resource that are in the market.

Taking in to account the measures of entrepreneurs both in and out of the market, the economy as a whole evolve in the following way.

For entrepreneurs that are “in-the-market” in period  $t$ , I have the following laws of motion for the wealth shares. First, the entrepreneurs that have productivity  $z'$

next period have wealth shares:

$$\omega_{t+1}(z') = \frac{K_t^{in} \int_0^\infty A(z, 1) \omega_t(z) \psi(z'|z) dz + K_t^{out} \int_0^\infty A(z, 1) u_t^0(z) \psi(z'|z) dz}{K_t^{in} \int_0^\infty A(z, 1) \omega_t(z) dz + K_t^{out} \int_0^\infty A(z, 1) u_t^0(z) dz} \quad (2.11)$$

For  $z > \underline{z}$ ,  $\phi_b(z)$  is an interior optimal choice depending on  $z$ , so the wealth share of the type  $(\phi_b(z), z')$  entrepreneurs is:

$$u_{t+1}^1(\phi_b(z), z') = \frac{K_t^{in} A(z, 1) \omega_t(z) \psi(z'|z) + K_t^{out} A(z, 1) u_t^0(z) \psi(z'|z)}{K_t^{in} \int_0^\infty A(z, 1) \omega_t(z) dz + K_t^{out} \int_0^\infty A(z, 1) u_t^0(z) dz} \quad (2.12)$$

For  $z \leq \underline{z}$ ,  $\phi_b(z) = -\beta$  and the wealth share of the type  $(-\beta, z')$  entrepreneurs is:

$$u_{t+1}^1(-\beta, z') = \frac{\int_0^{\underline{z}} K_t^{in} A(z, 1) \omega_t(z) \psi(z'|z) dz + \int_0^{\underline{z}} K_t^{out} A(z, 1) u_t^0(z) \psi(z'|z) dz}{K_t^{in} \int_0^\infty A(z, 1) \omega_t(z) dz + K_t^{out} \int_0^\infty A(z, 1) u_t^0(z) dz} \quad (2.13)$$

Finally, for entrepreneurs that have productivity  $z'$  next period and are “out-of-market” in period  $t$ , the wealth share is:

$$u_{t+1}^0(z') = \quad (2.14)$$

$$\frac{K_t^{in} \int_{-\beta}^\infty \int_0^\infty \{(\beta + \phi_b^-)(z\pi + 1 - \delta) - R_L \phi_b^-\} u_t^1(\phi_b^-, z) \psi(z'|z) dz d\phi_b^- + K_t^{out} \int_0^\infty \beta(z\pi + 1 - \delta) u_t^0(z) \psi(z'|z) dz}{K_t^{in} \int_{-\beta}^\infty \int_0^\infty \{(\beta + \phi_b^-)(z\pi + 1 - \delta) - R_L \phi_b^-\} u_t^1(\phi_b^-, z) dz d\phi_b^- + K_t^{out} \int_0^\infty \beta(z\pi + 1 - \delta) u_t^0(z) dz}$$

Given the current wealth shares  $\omega_t(z)$ ,  $u_t^1(\phi_b^-, z)$ ,  $u_t^0(z)$  and transition density  $\psi(z'|z)$  for type  $z$  entrepreneurs,  $\omega_{t+1}(z')$ ,  $u_{t+1}^1(\phi_b, z')$ ,  $u_{t+1}^0(z')$  keep track of the wealth shares of entrepreneurs at the beginning of period  $t + 1$ . Eventually, these wealth shares will stay unchanged as the system reach its stationary equilibrium.

From Lemma 1, the individual labor choice is  $l(a, b^-, z, s) = (\frac{1-\alpha}{W(X)})^{\frac{1}{\alpha}} z k$ , hence the labor market clearing condition for the economy is  $\int_0^\infty l(a, b^-, z, s) \omega(z) dz = \int_0^\infty (\frac{1-\alpha}{W(X)})^{\frac{1}{\alpha}} z k \omega(z) dz = L$ . In equilibrium the wage is determined by the distributions of entrepreneurs both in and out of the market. More specifically, the wage is obtained



by:

$$W_t = \frac{[(1-p) \int_{\bar{z}}^{\infty} \lambda z (K_t^{in} \omega_t(z) + K_t^{out} u_t^0(z)) dz + p \int_0^{\infty} z (K_t^{in} \int_0^{\infty} (1 + \frac{\phi_b^-}{\beta}) u_t^1(\phi_b^-, z) dz^- + K_t^{out} u_t^0(z)) dz]^{\alpha}}{(1-\alpha)^{-1} L^{\alpha}} \quad (2.15)$$

The short-term interest rate is determined by the marginal borrower, which has productivity  $\bar{z}$ . Since some entrepreneurs that are currently in the market may be out of the market in previous period, both the wealth shares of entrepreneurs in and out of the market matter. The degree of financial friction and the distribution of wealth shares of the entrepreneurs in the market will pin down  $\bar{z}$ . More specifically, I have

$$1 - \frac{1}{\lambda} = \int_0^{\bar{z}} \frac{K_t^{in} w_t(z) + K_t^{out} u_t^0(z)}{K_t^{in} + K_t^{out}} dz.$$

The long-term interest rate is determined by the market clearing condition for long-term debt. With wealth shares and long-term debt policy, the condition states  $\int_0^{\infty} \phi_b(z, R_L) \frac{K_t^{in} w_t(z) + K_t^{out} u_t^0(z)}{K_t^{in} + K_t^{out}} dz = 0$ . Finally, to find the total amount of resources in and out of the market, I have the following relations. For total resources in the market,  $K_{t+1}^{in} = (1-p) \{ K_t^{in} \int_0^{\infty} A(z, 1) \omega_t(z) dz + K_t^{out} \int_0^{\infty} A(z, 1) u_t^0(z) dz \}$ . For the resources out of the market,  $K_{t+1}^{out} = p \{ K_t^{in} \int_{-\beta}^{\infty} \int_0^{\infty} \{ (\beta + \phi_b^-)(z\pi + 1 - \delta) - R_L \phi_b^- \} u_t^1(\phi_b^-, z) dz d\phi_b^- + K_t^{out} \int_0^{\infty} \beta(z\pi + 1 - \delta) u_t^0(z) dz \}$ .

Now I investigate what affects the total use of long-term debt. From Proposition 19, I know that on the individual level, entrepreneurs with high (low) productivity will borrow (lend) more long-term debt as the productivity process becomes more persistent. Since both the demand and supply of long-term debt increase, the overall use of long-term debt must increase. Figure 2.4 illustrates the increase use of long-term debt for entrepreneurs with both low and high current productivities. The green solid line presents the long-term debt policy when TFP is more persistent ( $\rho = .9$ ). The teal dotted broken line presents the long-term debt policy when TFP is less persistent ( $\rho = .5$ ). Comparing the two line, we see that entrepreneurs with TFP below 1.36 choose to reduce long-term debt and entrepreneurs with TFP above

1.36 choose to increase long-term debt as TFP becomes more persistent. Suppose that the wealth share of the those entrepreneurs stay the same after the increase in persistence, then there are must be more use of long-term debt when TFP becomes more persistence. In reality, the wealth shares of entrepreneurs with high TFP will increase compared to those with low TFP. Moreover, the equilibrium long-term debt interest rate will change as well. Hence, I can only state the intuition for the increase use of long-term debt without a formal proposition.

When the productivity shocks are more persistent, the entrepreneurs can predict future productivities better from the current productivity. Therefore, the currently productive entrepreneurs are willing to borrow more in long-term debt and the currently unproductive entrepreneurs are willing to lend more in long-term debt. As a result, they are both better off when they are out of the market in the future.

## 2.5 Debt Maturity and TFP losses

### 2.5.1 *Importance of long-term debt*

From the model we see that the predictability of TFP affects debt maturity in the economy. In particular, when TFP is more predictable, more long-term debt will be used in the economy. As illustrated in Figure 2.4, the entrepreneurs with high  $z$  increase long-term debt borrowing and the entrepreneurs with low  $z$  increase long-term debt lending as the persistence parameter  $\rho$  increases from 0.5 to 0.9.

However, debt maturity also has impact on aggregate TFP. When TFP is persistent, the currently more productive entrepreneurs are more likely to stay productive next period. Hence, if the productive entrepreneurs borrowed long-term debt currently, there is a good chance that they will need the borrowed resources still next period. Therefore, having long-term debt will alleviate capital misallocation when the entrepreneurs are out of the market next period. Among all entrepreneurs that are out of the market, the active entrepreneurs who produces are the productive ones

and they have more resources. Hence, the average productivity for out of the market entrepreneurs increases.

Moreover, when the TFP is more persistent, the more productive entrepreneurs will accumulate more wealth and stay productive more often. Hence, the stationary distribution of the entrepreneurs will have a first order stochastic dominant improvement. That is more resources will be in the hands of the more productive entrepreneurs. This point is illustrated in Figure 2.4, where more weights are shifted to the more productive entrepreneurs in the wealth shares of “in-the-market” entrepreneurs after  $\rho$  increases from 0.5 to 0.9. In fact, as show in Figure 2.5, the wealth shares for both in and out of the market entrepreneurs increase for the more productive entrepreneurs. Both this direct improvement of the wealth shares and the indirect reduction of misallocation for “out-of-market” contribute to a higher aggregate TFP and growth rate of capital. Therefore, we have the following proposition.

**Proposition 21.** *Aggregate (average) productivity increases as the idiosyncratic TFP process becomes more persistent.*

Moreover, the effect of long-term debt is bigger when TFP is more persistent. To illustrate the point, I compare the average growth rate of capital of two groups of entrepreneurs that are out of the market. One group comes into the period with existing long-term debt optimally chosen in last period. The other comes into the period with no long-term debt because it was out of the market in last period as well. The growth rate of capital is the return from capital net of wage and interest. Table 2.2 shows that the average growth rate of capital of entrepreneurs without long-term debt is 6.79% lower than that of entrepreneurs with long-term debt when TFP is very persistent ( $\rho = .9$ ). However, the average growth rate of capital of entrepreneurs without long-term debt is only 3.66% lower than that of entrepreneurs with long-term debt when TFP is less persistent ( $\rho = .5$ ). Hence, long-term debt plays a more important role when TFP is more persistent.

### 2.5.2 *Pro-cyclical Debt Maturity*

As mentioned before, Bloom and others have documented a counter-cyclical uncertainty in productivity. I argue that there is also a pro-cyclical persistence which contributes to blurry future prospects during recessions. From a quarterly data of Compustat from 1973 to 2012, I estimate the persistence parameter. Since TFP is hard to measure with Compustat data due to limited information on wage expense, I look at the persistence in sales. I divide the data into ten subperiod datasets according to NBER recession dates. So, I have five expansion periods and five recession period. Then for each period, I run an AR(1) regression on the log of sales for each firm. Then I take an equal-weighted average over all the firms. From the results shown in Table 2.1, we see that indeed sales are much more persistence during expansions (the second row from Panel A) than during recessions (the first row from Panel A) for all periods. In fact, on average sales is about three times more persistent during expansions than during recessions. At the same time, debt maturity is longer during expansions than during recessions. From Panel B of Table 2.1, the fraction of debt maturing in a year is consistently higher during expansions than during recessions, with the exception of recession from January 1980 to November 1982. Also in Panel C, the value weighted maturity of all debt recorded in the DealScan data is clearly higher in during expansions than during recessions.

Hence, the empirical pattern seems to be consistent with the story that the pro-cyclical debt maturity is caused by the pro-cyclical persistence in productivity. Next, I examine the difference between the growth rates of capital for in and out of market entrepreneurs. I investigate how the difference changes from expansion to recession and how those differences change when the probability of being excluded from the market rises.

Since expansions are associated with higher persistence and lower volatility, I set the persistence parameter  $\rho$  to be .9 (.5) and standard deviation of error term  $\sigma$  to

be .3775 (.75) for expansion (recession). As shown in Table 2.3, During recession, there is a much larger reduction in the growth rate of capital from in the market to out of the market. This is a direct result of the reduced use of long-term debt during recessions. The growth rate of capital for “out-of-market” entrepreneurs in recession is only about 1.05 compared to about 1.08 during expansion. Although the reduction in growth rate is across the board higher in recession than that in expansion, during expansion the reduction is more sensitive to the probability of being excluded from the market. Basically, two things happen when the probability of out of the market increases. First, since a larger fraction of the total resources will grow at a lower “out-of-market” growth rate, the wage has to decrease so that both in the market and out of market growth rates will increase. Second, among all entrepreneurs that are out of the market, a larger fraction of them come into the period without long-term debt. Hence, a larger fraction of them will grow at a lower “no long-term debt” rate as illustrated in Table 2.2. Hence, the average growth rate for all “out-of-market” entrepreneurs should decrease. During expansion the second effect dominates the first since the difference between “with long-term debt” and “no long-term debt” growth rate is much bigger than that during recession (as illustrated in Table 2.2). Hence, during expansion, the reduction in growth rate increases dramatically as the probability of out of the market increases. Therefore, a financial crisis that suddenly makes everyone difficult to borrow may have bigger impact on aggregate growth rate during expansions.

## 2.6 Conclusion

I introduce a model that illustrates how counter-cyclical uncertainty and pro-cyclical persistence in productivity lead to pro-cyclical aggregate debt maturity. In the model, both short-term and long-term debt are vehicles through which the entrepreneurs transfer resources from one to another. In each period, before production takes

place, the more productivity entrepreneurs will borrow from the less productivity ones. Hence, the resources will be put into more efficient use. However, when the entrepreneurs are sometimes unable to conduct such transfer in future periods, they arrange for long-term transfer so that the productive ones may have more resources when they can't borrow in the future. Hence, the amount of long-term debt used is directly linked to how predictable the productivity process is. When TFP is highly persistent, more long-term debt is used. Long-term debt plays an important role in alleviating capital misallocation and preserving aggregate productivity and growth rate of capital. Finally, a sudden increase in the probability of not being able to borrow may have a greater impact on aggregate growth rate during expansion than during recession.

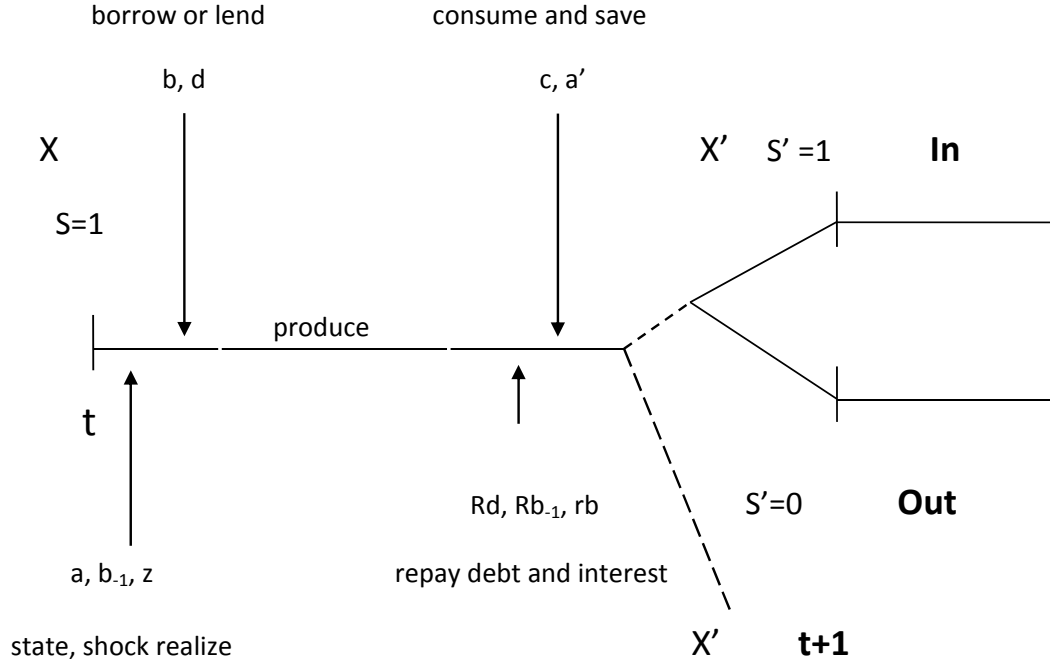


FIGURE 2.1: **The Entrepreneur's Problem (Timeline).**

This figure presents the timeline when the entrepreneur is currently “in-the-market” ( $s=1$ ). The current aggregate state which include the entire distribution of the entrepreneurs is summarized by  $X$ . Similarly, next period aggregate state is summarized by  $X'$ . In the next period, the entrepreneur can either be “in-the-market” ( $s'=1$ ) or “out-of-market” ( $s'=0$ ). The idiosyncratic states are net worth,  $a$ , existing long-term debt,  $b^-$ , and productivity,  $z$ . The entrepreneur chooses, new levels of long-term debt,  $b$ , and short-term debt,  $d$  before production. The entrepreneur chooses, consumption,  $c$ , and end period net worth  $a'$ . The entrepreneur also repays interest and principle on current short-term debt,  $Rd$ , and on pre-existing long-term debt,  $Rb_{-1}$ , and interest on new long-term debt,  $rb$ .

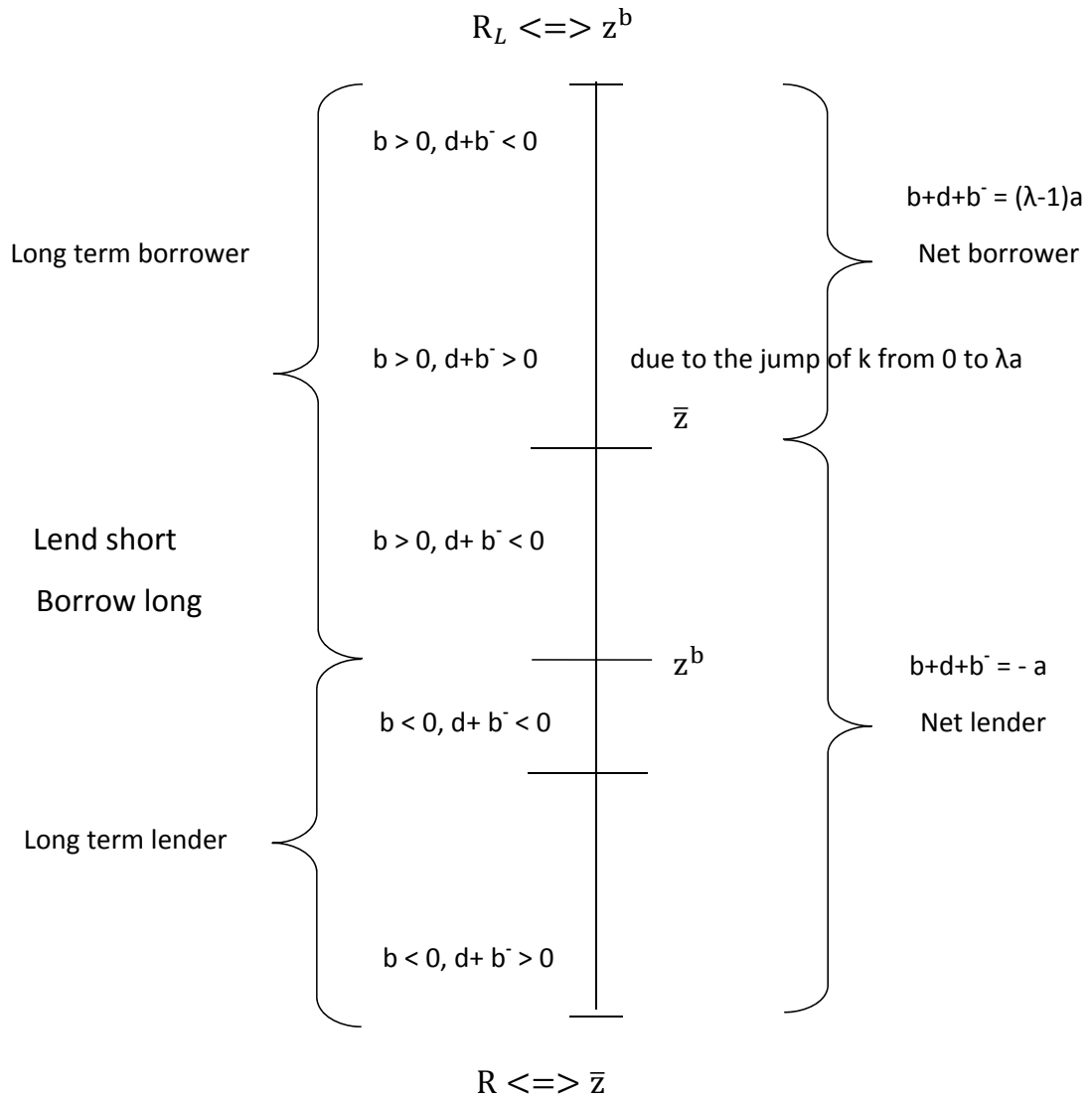


FIGURE 2.2: **Spectrum of Entrepreneurs in the Market.**

This figure summarizes the optimal choices by entrepreneurs across different current productivity levels. As we move upwards, current productivity realization increases. Entrepreneurs with productivity above  $\bar{z}$  are net borrowers. Entrepreneurs with productivity below  $\bar{z}$  are net lenders. Entrepreneurs with productivity above  $z^b$  are long-term borrowers. Entrepreneurs with productivity below  $z^b$  are long-term lenders.



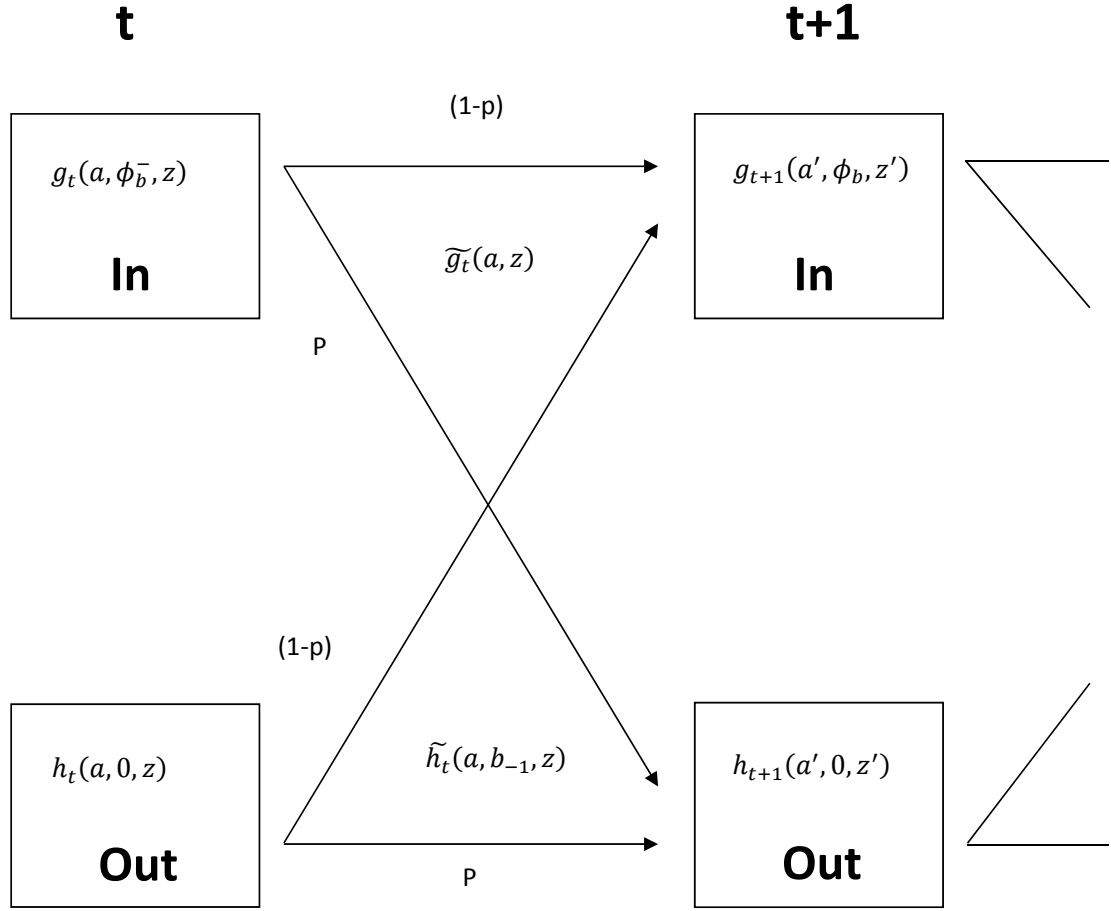


FIGURE 2.3: **Evolution of the Economy.**

This figure illustrates how the economy evolves from one period to the next. The function  $g_t(a, \phi_b^-, z)$  keeps track of the end of period  $t - 1$  net worth, long-term debt, and new productivity of the entrepreneurs who were previously “in-the-market”. The function  $h_t(a, 0, z)$  keeps track of the end of period  $t - 1$  net worth, long-term debt, and new productivity of the entrepreneurs who were previously “out-of-market”. In period  $t$ , a fraction  $p$  of the entrepreneurs goes “out-of-market” and a fraction  $1 - p$  of the entrepreneurs goes “in-the-market”. Prior to production, the currently “in-the-market” entrepreneurs are identified by their net worth  $a$  and productivity  $z$ . The function  $\tilde{g}_t(a, z)$  is the measure of those entrepreneurs. On the other hand, the currently “out-of-market” entrepreneurs are identified by their net worth  $a$ , productivity  $z$  and existing long-term debt,  $b^-$ . The function  $\tilde{h}_t(a, b^-, z)$  is the measure of those entrepreneurs. At the end of period  $t$ , the distributions  $g_{t+1}(a', \phi_b, z')$  and  $h_{t+1}(a', 0, z')$  are obtained from  $\tilde{g}_t(a, z)$  and  $\tilde{h}_t(a, b^-, z)$  after taking into account of all individual decisions in period  $t$ .

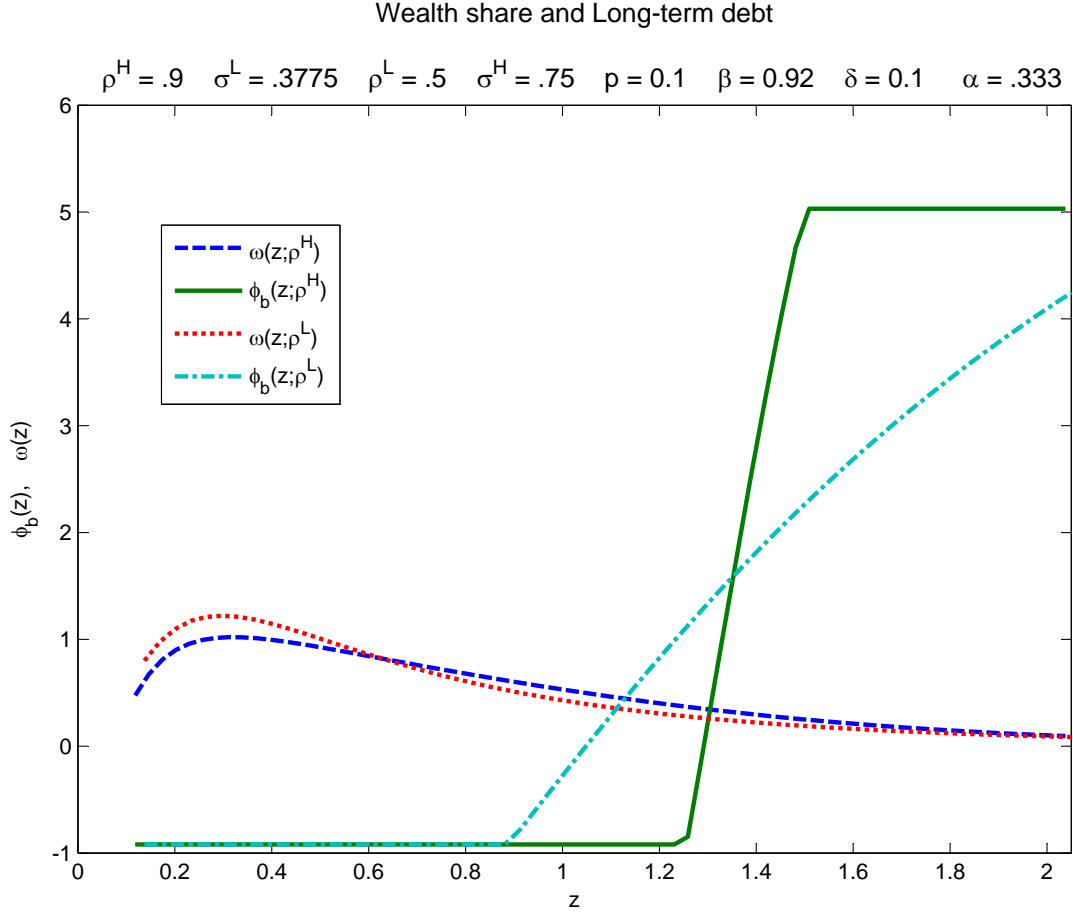


FIGURE 2.4: **Wealth Shares and Long-term Debt Policy.**

This figure plots the “in-the-market” wealth shares  $\omega(z)$  and the long-term debt policy function  $\phi_b(z)$  for two different levels of persistence and volatility of TFP. The pair  $(\rho^H = .9, \sigma^L = .3775)$  represents economic expansions in which the TFP is more predictable. The pair  $(\rho^L = .5, \sigma^H = .75)$  represents economic recessions in which the TFP is less predictable.  $\rho^H$  ( $\rho^L$ ) is used to indicate expansion (recession). The green solid line is long-term debt policy in expansion. The blue broken line is the corresponding “in-the-market” wealth shares at the steady state in expansion. The teal dotted broken line is the long-term debt policy in recession. The red dotted line is the corresponding “in-the-market” wealth shares in recession.

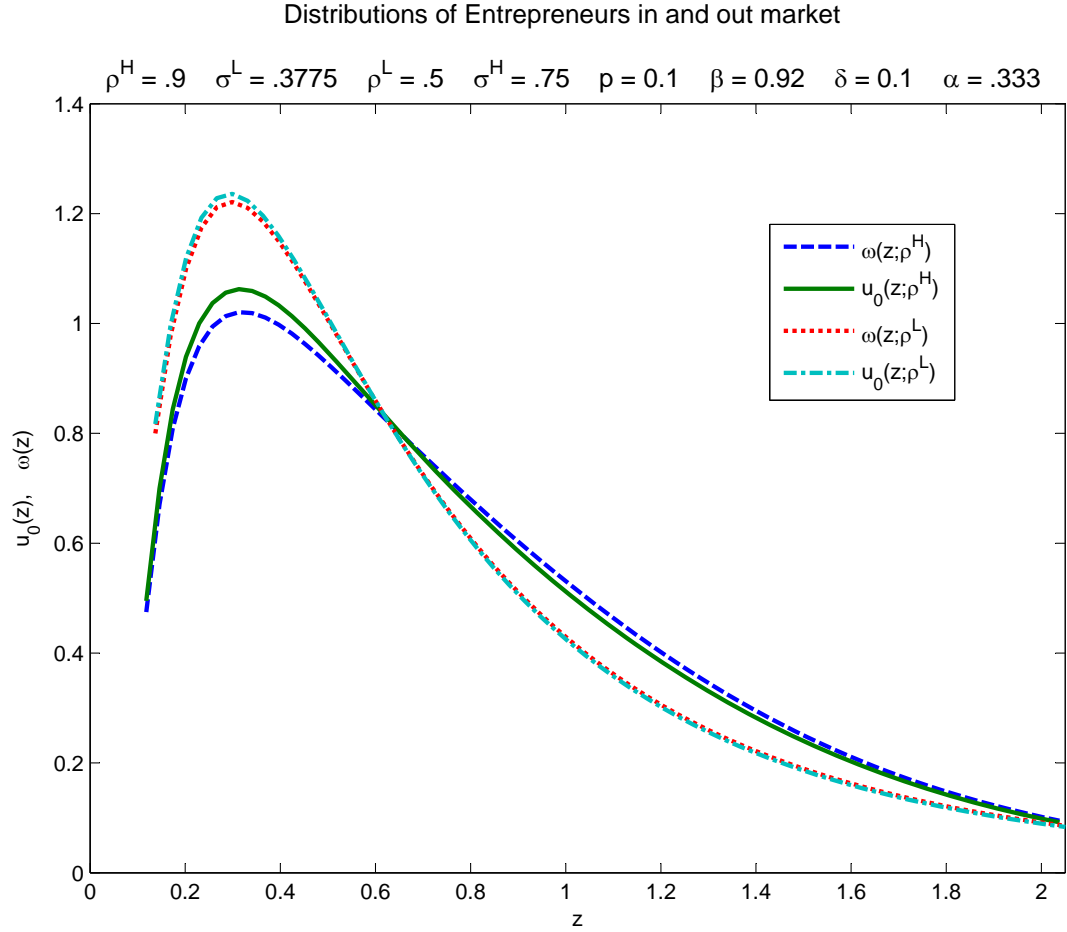


FIGURE 2.5: **Wealth Shares In and Out of Market.**

This figure plots the “in-the-market” wealth shares  $\omega(z)$  and “out-of-market” wealth shares  $u_0(z)$  for two different levels of persistence and volatility of TFP. The pair  $(\rho^H = .9, \sigma^L = .3775)$  represents economic expansions in which the TFP is more predictable. The pair  $(\rho^L = .5, \sigma^H = .75)$  represents economic recessions in which the TFP is less predictable.  $\rho^H$  ( $\rho^L$ ) is used to indicate expansion (recession). The green solid line is “out-of-market” wealth shares in expansion. The blue broken line is the corresponding “in-the-market” wealth shares at the steady state in expansion. The teal dotted broken line is “out-of-market” wealth shares in recession. The red dotted line is the corresponding “in-the-market” wealth shares in recession.

Table 2.1: **Persistence of Idiosyncratic Productivity.**

This table presents the persistence of TFP and the average debt maturity for different subperiods. The persistence parameters shown in panel A are estimated from an AR(1) regression of the natural log of sales from Compustat Quarterly Files. For the regressions for recession periods (shown on the first line), I also included one quarter before and after the NBER recession. The debt maturity shown in panel B is measured as the fraction of debt in current liability out of total debt. Total debt is the sum of debt in current liability and long-term debt. the data are from Compustat Quarterly Files. The debt maturity shown in panel C is measured in month. The data is from DealScan. The persistence parameters are equal weighted averages of firm persistence parameters. The debt maturity measures are value weighted averages, hence represent aggregate debt maturity.

Panel A : Subperiod Productivity Persistence				
Nov73-Mar75 0.3411	Jan80-Nov82 0.2896	Jul90-Mar91 -0.0610	Mar01-Nov01 0.1607	Dec07-Jun09 0.2945
Apr75-Dec79 0.6665	Dec82-Jun90 0.5249	Apr91-Feb01 0.5835	Dec01-Nov07 0.5388	Jul09-Dec12 0.4052
Panel B : Subperiod Debt Maturity (Compustat)				
Nov73-Mar75 0.669	Jan80-Nov82 0.697	Jul90-Mar91 0.629	Mar01-Nov01 0.627	Dec07-Jun09 0.681
Apr75-Dec79 0.722	Dec82-Jun90 0.654	Apr91-Feb01 0.643	Dec01-Nov07 0.670	Jul09-Dec12 0.693
Panel C : Subperiod Debt Maturity (DealScan)				
	Jul90-Mar91 52	Mar01-Nov01 32	Dec07-Jun09 47	
Jul86-Jun90 57	Apr91-Feb01 45	Dec01-Nov07 49	Jul09-Dec12 53	

Table 2.2: **Effects of Long-term Debt.**

This table presents the steady state aggregate growth rates of capital for “out-of-market” entrepreneurs with or without long-term debt under two types of economic conditions. There are two types of “out-of-market” entrepreneurs. The first type “With Long-term Debt” means the entrepreneurs can choose the optimal long-term debt last period. The first type “Without Long-term Debt” means the entrepreneurs cannot choose the optimal long-term debt last period. Hence, they have zero long-term debt. The aggregate growth rate is the wealth weighted average of individual growth rates. The reduction in growth is the difference between with and without long-term debt growth rate scaled by with long-term debt growth rate. Expansions correspond to a setup with high persistence  $\rho = .9$  and low volatility  $\sigma = .3775$  of productivity. Recessions correspond to a setup with low persistence  $\rho = .5$  and high volatility  $\sigma = .75$  of productivity.

	Recessions ( $\rho = .5$ $\sigma = .75$ )	Expansions ( $\rho = .9$ $\sigma = .3775$ )
With Long-term Debt	1.0528	1.0875
No long-term Debt	1.0143	1.0137
Reduction in growth	3.66%	6.79%

Table 2.3: **Growth for In and Out of the Market.**

This table presents the steady state aggregate growth rates of capital for “In-the-market” and “out-of-market” entrepreneurs and steady state fraction of resources in the hands of “out-of-market” entrepreneurs across different probability of “out-of-market”. The first two rows show the aggregate growth rates for “In-the-market” and “out-of-market” entrepreneurs. The aggregate growth rate is the wealth weighted average of individual growth rates. The reduction in growth is the difference between in and out of market growth rates scaled by in the market growth rate. The resource out of the market is the fraction of total resource that are in the hand of “out-of-market” entrepreneurs in steady state. As we move from left to right, the probability of out of the market increase from 0.05 to 0.4. Expansions correspond to a setup with high persistence  $\rho = .9$  and low volatility  $\sigma = .3775$  of productivity. Recessions correspond to a setup with low persistence  $\rho = .5$  and high volatility  $\sigma = .75$  of productivity.

	$p = 0.05$	$p = 0.1$	$p = 0.2$	$p = 0.4$
Panel A: in recessions ( $\rho = .5$ $\sigma = .75$ )				
In the Market	1.0889	1.0911	1.0962	1.1103
Out of Market	1.0497	1.0494	1.0499	1.0520
Reduction in growth	3.60%	3.82%	4.22%	5.25%
Resource out of Market	4.83%	9.65%	19.32%	38.71%
Panel B: in expansions ( $\rho = .9$ $\sigma = .3775$ )				
In the Market	1.0870	1.0877	1.0899	1.1017
Out of Market	1.0861	1.0802	1.0752	1.0645
Reduction in growth	0.08%	0.69%	1.34%	3.38%
Resource out of Market	4.97%	9.95%	19.98%	39.56%

# Appendix A

## Proofs

### Appendix A: Value function properties

*Proof of Proposition 1.* Following Rampini and Viswanathan (2010), I prove the proposition in five parts below. To simplify the notation, I use  $b^-$  and  $b$  instead of  $b_L^-$  and  $b_L$  to represent current and next period long-term debt. Also, I will use  $\pi(s')$  so that  $\pi(0) = \pi$  and  $\pi(1) = 1 - \pi$ .

(i)  $\Gamma(w, b_L^-, z, s)$  is a convex set, given  $(w, b_L^-, z, s)$ , and (jointly) convex in  $w$  and  $b^-$  given  $z$  and  $s$ .

**Proof of (i).** Suppose  $x, \hat{x} \in \Gamma(w, b^-, z, s)$ . For  $\psi \in (0, 1)$ , let  $x_\psi = \psi x + (1 - \psi)\hat{x}$ . Then  $x_\psi \in \Gamma(w, b^-, z, s)$  because equations (1.3), (1.5), (1.6), and (1.7) are linear and because  $A(z')f(k)$  is concave in  $k$ .

Now let  $x \in \Gamma(w, b^-, z, s)$  and  $\hat{x} \in \Gamma(\hat{w}, \hat{b}^-, z, s)$ . For  $\psi \in (0, 1)$ , let  $x_\psi = \psi x + (1 - \psi)\hat{x}$ . Since  $\Gamma(w, b^-, z, s)$  is convex given  $w, b^-, z, s$  and since equations (1.4), (1.5), and (1.7) do not involve  $w$  or  $b^-$ , I only need to check equations (1.3) and (1.6). Since

both constraints are linear in  $x$ ,  $b^-$ , and  $w$ , I have:

$$\psi(w + b_S + b) + (1 - \psi)(w + b_S + b) \geq \psi(d + k) + (1 - \psi)(d + k)$$

$$\psi b^- + (1 - \psi)\hat{b}^- \geq \psi b + (1 - \psi)\hat{b}, \quad \text{for } s = 0$$

Thus,  $x_\psi \in \Gamma(\psi w + (1 - \psi)\hat{w}, \psi b^- + (1 - \psi)\hat{b}^-, z, s)$ . Therefore,  $\Gamma(w, b^-, z, s)$  is jointly convex in  $w$  and  $b$ .

**Lemma 2**  $\Gamma(w, b^-, z, s)$  is monotone in  $w$  in the sense that  $w \leq \hat{w}$  implies  $\Gamma(w, b^-, z, s) \subseteq \Gamma(\hat{w}, b^-, z, s)$  and  $\Gamma(w, b^-, z, s)$  is monotone in  $b^-$  in the sense that  $b^- \leq \hat{b}^-$  implies  $\Gamma(w, b^-, z, s) \subseteq \Gamma(w, \hat{b}^-, z, s)$ .

**Proof of Lemma 2.** Given  $b^-$ ,  $z$ , and  $s$ , increasing  $w$  only relaxes constraint (1.3). So, any feasible choice under  $\Gamma(w, b^-, z, s)$  is still feasible under  $\Gamma(\hat{w}, b^-, z, s)$ . Thus, if  $w \leq \hat{w}$ , then  $\Gamma(w, b^-, z, s) \subseteq \Gamma(\hat{w}, b^-, z, s)$ .

Similarly, given  $w$ ,  $z$ , and  $s$ , increasing  $b^-$  only relaxes constraint (1.6). So, if  $b^- \leq \hat{b}^-$ , then  $\Gamma(w, b^-, z, s) \subseteq \Gamma(w, \hat{b}^-, z, s)$ .

**Lemma 3** The operator  $T$  satisfies Blackwell's sufficient conditions for a contraction and has a unique fixed point  $V$ .

**Proof of Lemma 3.** Suppose  $g(w, b^-, z, s) \geq f(w, b^-, z, s), \forall (w, b^-, z, s) \in \mathbb{R}_+^2 \times \mathbb{R} \times \{0, 1\}$ . Then, for any  $x \in \Gamma(w, b^-, z, s)$ ,

$$\begin{aligned} (Tg)(w, b^-, z, s) &\geq d + \beta \sum_{s'} \pi(s') \int g(w(z'), b, z', s') dQ_z \\ &\geq d + \beta \sum_{s'} \pi(s') \int f(w(z'), b, z', s') dQ_z. \end{aligned}$$

Thus,

$$(Tg)(w, b^-, z, s) = \max_{x \in \Gamma(w, b^-, z, s)} d + \beta \sum_{s'} \pi(s') \int g(w(z'), b, z', s') dQ_z$$



$$\geq \max_{x \in \Gamma(w, b_L^-, z, s)} d + \beta \sum_{s'} \pi(s') \int f(w(z'), b, z', s') dQ_z = (Tf)(w, b^-, z, s)$$

for all  $(w, b^-, z, s) \in \mathfrak{R}_+^2 \times \mathfrak{R} \times \{0, 1\}$ . Thus,  $T$  satisfies monotonicity.

Also,

$$\begin{aligned} T(f + a)(w, b^-, z, s) &\leq \max_{x \in \Gamma(w, b^-, z, s)} d + \beta \sum_{s'} \pi(s') \int (f + a)(w(z'), b, z', s') dQ_z \\ &= (Tf)(w, b^-, z, s) + \beta a \end{aligned}$$

Therefore,  $T$  satisfies discounting. Hence,  $T$  is a contraction and has a unique fixed point  $V$  by the contraction mapping theorem.

**Lemma 4**  $V$  is continuous, strictly increasing in  $w$ .  $V$  is continuous, increasing in  $b^-$ . And  $V$  is jointly concave in  $w$  and  $b^-$ .

**Proof of Lemma 4.** Let  $x_o \in \Gamma(w, b^-, z, s)$  and  $\hat{x}_o \in \Gamma(\hat{w}, \hat{b}^-, z, s)$  be the optimal choices that attain  $(Tf)(w, b^-, z, s)$  and  $(Tf)(\hat{w}, \hat{b}^-, z, s)$  respectively.

I first prove the joint concavity of  $V$  in  $w, b^-$ , given  $z$  and  $s$ .

Suppose  $f$  is jointly concave in  $w, b^-$ . Then, for  $\psi \in (0, 1)$ , let  $x_{o,\psi} \equiv \psi x_o + (1-\psi)\hat{x}_o$  and  $w_\psi = \psi w + (1-\psi)\hat{w}$ ,  $b_\psi^- = \psi b^- + (1-\psi)\hat{b}^-$ . I have

$$\begin{aligned} (Tf)(w_\psi, b_\psi^-, z, s) &\geq d_{o,\psi} + \beta \sum_{s'} \pi(s') \int f(w_{o,\psi}(z'), b_{o,\psi}, z', s') dQ_z \\ &\geq \psi [d_o + \beta \sum_{s'} \pi(s') \int f(w_o(z'), b_o, z', s') dQ_z] \\ &\quad + (1-\psi) [\hat{d}_o + \beta \sum_{s'} \pi(s') \int f(\hat{w}_o(z'), \hat{b}_o, z', s') dQ_z] \\ &= \psi (Tf)(w, b^-, z, s) + (1-\psi) (Tf)(\hat{w}, \hat{b}^-, z, s). \end{aligned}$$

The first inequality is by the fact that  $\Gamma(w, b, z, s)$  is jointly convex in  $w$  and  $b$ .

So,  $x_{o,\psi} \in \Gamma(w_\psi, b_\psi, z, s)$ . The second inequality is by the fact  $f$  is assumed to be jointly concave.

Thus,  $Tf$  is jointly concave in  $w$  and  $b$ .  $T$  maps jointly concave functions into jointly concave functions. Therefore,  $V$  is jointly concave in  $w$  and  $b$ . Hence, the first order conditions are also sufficient conditions; and  $V$  is also concave in  $w$  and concave in  $b$ .

Suppose  $f$  is increasing in  $b$  and  $b \leq \hat{b}$ .

$$\begin{aligned} (Tf)(w, \hat{b}^-, z, s) &= \hat{d}_o + \beta \sum_{s'} \pi(s') \int f(w(z'), \hat{b}_o, z', s') dQ_z \\ &\geq d + \beta \sum_{s'} \pi(s') \int f(w(z'), b, z', s') dQ_z \end{aligned}$$

Hence,

$$(Tf)(w, \hat{b}^-, z, s) \geq \max_{x \in \Gamma(w, b^-, z, s)} d + \beta \sum_{s'} \pi(s') \int f(w(z'), b, z', s') dQ_z = (Tf)(w, b^-, z, s)$$

So,  $Tf$  is increasing in  $b$ .  $T$  maps increasing functions into increasing functions. Therefore,  $V$  is increasing in  $b$ .

Similar arguments using  $w$  and  $\hat{w}$  show that  $V$  is increasing in  $w$ . Plus, suppose  $w < \hat{w}$ , then

$$(Tf)(\hat{w}, b^-, z, s) \geq (\hat{w} - w) + d_o + \beta \sum_{s'} \pi(s') \int f(w_o(z'), b, z', s') dQ_z > (Tf)(w, b^-, z, s)$$

So,  $Tf$  is strictly increasing in  $w$ .  $T$  maps increasing functions into strictly increasing functions. Therefore,  $V$  is strictly increasing in  $w$ .

Finally, since  $V$  is both increasing and concave in  $w$  and in  $b^-$ ,  $V$  is continuous in both  $w$  and  $b^-$ .

**Lemma 5**  $V(w, b^-, z, s)$  is strictly concave in  $w$  for  $w \in \text{int}\{w : d(w, b^-, z, s) = 0\}$ .  $V(w, b^-, 1, 0)$  is strictly concave in  $w$  and  $b^-$  for  $\{w, b^-\} \in \text{int}\{\{w, b^-\} : d(w, b^-, z, 0) = 0, b^- < \frac{1-p_L}{p_L}w\}$ .

**Proof of Lemma 5.** First I show that suppose for  $\forall w, \hat{w} \in \text{int}\{w : d(w, b^-, z, s) = 0\}$  and  $\hat{w} \geq w$ , if the optimal choices is such that  $\hat{k}_o \neq k_o$ , then I have for  $\psi \in (0, 1)$  and  $x_{o,\psi} \equiv \psi x_o + (1-\psi)\hat{x}_o$ ,  $V(w_\psi, b^-, z, s) > d_{o,\psi} + \beta \sum_{s'} \pi(s') \int V(w_{o,\psi}(z'), b_{o,\psi}, z', s') dQ_z \geq \psi V(w, b^-, z, s) + (1-\psi)V(\hat{w}, \hat{b}^-, z, s)$ . The strict inequality is due to the fact that constraint (1.4) is slack hence net worth levels  $w(z') > w_{o,\psi}(z')$  are feasible for all  $z'$ . The second inequality is due to the concavity of  $V$ . For simplicity, I let  $p_L = 1 - R_L^{-1}\theta(1-\delta)$  represents the minimal down payment as discussed in Section 6.

From the FOCs, we have that when  $w \rightarrow 0$ ,  $A(z')f_k(k) + (1-\delta) > R_L$ ,  $\forall z' \in Z$ . Hence, there exists a level of net worth  $\tilde{w}$  s.t.  $\forall w < \tilde{w}$ , either the collateral constraint is binding  $\lambda > 0$  or the financing constraint is bind  $\gamma_L > 0$ . Then  $\forall w < \hat{w} < \tilde{w}$  and while  $\lambda > 0$ ,  $\hat{k}_o > k_o$ . Hence,  $V(w, b^-, z, 0)$  with  $b^- > \frac{1-p_L}{p_L}w$  and  $V(w, b^-, z, 1)$  are strictly concave in  $w$  for  $w < \tilde{w}$ . Also, for  $s = 0$ , given some  $b^-, \hat{b}^- \leq \frac{1-p_L}{p_L}\tilde{w}$ , and  $w, \hat{w} < \tilde{w}$ , such that  $\{w, b^-\} \neq \{\hat{w}, \hat{b}^-\}$ , it must be that  $k_o > \hat{k}_o$  because  $\hat{b}_o = \hat{b}^-$ ,  $b_o = b^-$ ,  $d_o = \hat{d}_o = 0$ ,  $b_{S,o} = \hat{b}_{S,o} = 0$ , and budget constraints (1.3) and (1.4) are always binding. Thus,  $V$  is strictly jointly concave in  $w$  and  $b^-$ ,  $\forall \{w, b^-\} \in (0, \tilde{w}] \times [0, \frac{1-p_L}{p_L}\tilde{w}]$ .

Now suppose  $w, \hat{w} \geq \tilde{w}$ ,  $\hat{w} > w$ , and  $\hat{k}_o = k_o$ . Then,  $\hat{b} + \hat{b}_S < b + b_S$  by equation (1.3). Hence, either  $\hat{w}(z') > w(z')$  for all  $z'$  or  $\hat{b} > b$ . Then for some low  $z'$ ,  $\hat{w}(z'), w(z') < \tilde{w}$ , and for those  $z'$  states,  $V(w_{o,\psi}(z'), b_{o,\psi}, z', 0) > \psi V(w_o(z'), b_o, z, 0) + (1-\psi)V(\hat{w}_o(z'), \hat{b}_o, z, 0)$ . Thus,  $V(w_\psi, b^-, z, s) > \psi V(w, b^-, z, s) + (1-\psi)V(\hat{w}, \hat{b}^-, z, s)$ , and  $V$  is also strictly concave for  $\tilde{w} < w < \hat{w}$ . Next, let  $\hat{w}$  be the new  $\tilde{w}$ , and applying the same logic, I have that for all  $w \in \text{int}\{w : d(w, b^-, z, s) = 0\}$ ,  $V$  is strictly concave in  $w$ .

As asserted in Result 3, when  $s = 0$ , and  $w > \frac{p_L}{1-p_L}b^-$ , then  $b_o = b^-$ . Hence,

for  $\{w, b^-\}, \{\hat{w}, \hat{b}^-\} \in (0, w] \times [0, \frac{1-p_L}{p_L}w)$  and  $\{w, b^-\} \neq \{\hat{w}, \hat{b}^-\}$ , then either  $k_o \neq \hat{k}_o$  or  $w_o(z') \neq \hat{w}_o(z') \forall z'$  because equations (1.3) and (1.4) hold with equalities and since  $b_o = b^-, \hat{b}_o = \hat{b}^-, d_o = \hat{d}_o = 0$ , and  $R_L > R_S$ . If the later is true, then  $\exists w_o(z') \neq \hat{w}_o(z')$ , and  $w_o(z'), \hat{w}_o(z') \in \text{int}\{w : d(w, b^-, z, s) = 0\}$  for some  $z'$ . Then a convex combination can strictly increase  $V(w(z'), b, z', s')$  for those  $z'$ . Therefore,  $V$  is strictly (jointly) concave in  $w$  and  $b^-, \forall \{w, b^-\} \in (0, w] \times [0, \frac{1-p_L}{p_L}w)$  and where  $d_o = 0$ .

**Lemma 6** Since the debts are non-state contingent and collateralized, if  $k > 0$ , then  $w(z') > 0 \forall z' \in Z$ . More formally, from equation (1.4) and constraint (1.5), we have:  $w(z') \geq A(z')f(k) + (1 - \delta)(1 - \theta)k > 0 \forall z' \in Z$ .

Now to show that  $k > 0$ , note that  $k \rightarrow 0, A(z')f_k(k) \rightarrow \infty \forall z'$  by Assumptions 1 and 2. Thus, for  $k$  sufficiently small,  $A(z')f_k(k) + (1 - \delta) > R_L > R_S \forall z' \in Z$ . Then from equations (1.9) and (1.11), I obtain that when  $s = 1, \lambda > 0$ . Similar for  $s = 0$ , for any given  $b_L$  if  $w$  is sufficiently low, constraint (1.6) will not bind and  $\lambda > 0$ . From constraint (1.3),  $w + b_S + b_L \geq k$ . Thus, for some  $w > 0$ , constraint (1.3) will bind only if  $k > 0$ . Therefore starting with some  $w > 0$ , all subsequent  $w(\{z, s\}^t)$  and  $k(\{z, s\}^t)$  are positive.  $\square$

## Appendix B: Policy Functions

*Proof of Proposition 2.* Part (i): From equation 1.8,  $V_w(w, b^-, z, s) = \mu(w, b^-, z, s) \geq 1$ . Since  $V$  is concave in  $w$  (Lemma 4),  $\mu(w, b^-, z, s)$  is decreasing in  $w$ . If  $d(\hat{w}, b^-, z, s) > 0$ , then  $\mu(\hat{w}, b^-, z, s) = 1$  and  $\mu(w, b^-, z, s) = 1$  for all  $w \geq \hat{w}$ . Let  $\bar{w}(b^-, z, s) = \inf\{w : d(w, b^-, z, s) > 0\}$ .

Part (ii): Suppose  $w > \hat{w} \geq \bar{w}(b^-, z, s)$  and let  $\hat{x}_o$  attain  $V(\hat{w}, b^-, z, s)$ .

$V(w, b^-, z, s) = V(\hat{w}, b^-, z, s) + \int_{\hat{w}}^w dV$ , since  $V_1(w, b^-, z, s) = 1$  for any  $w \geq \bar{w}(b^-, z, s)$ . Hence  $x_o = [w - \bar{w} + \hat{d}_o, \hat{k}_o, \hat{w}_o(z'), \hat{b}_o]$  attains  $V(w, b^-, z, s)$ .

Suppose  $\hat{x}_o, \tilde{x}_o$  both attain  $V(\hat{w}, b^-, z, s)$ , and  $\hat{k}_o \neq \tilde{k}_o$ . Then by the fact that  $\Gamma(w, b^-, z, s)$  is a convex set and the fact that  $f(k)$  is strictly concave,

$$Af(k_{o,\psi}) + (1 - \delta)k_{o,\psi} > \psi[Af(\hat{k}_o) + (1 - \delta)\hat{k}_o] + (1 - \psi)[Af(\tilde{k}_o) + (1 - \delta)\tilde{k}_o]$$

where  $k_{o,\psi} = \psi\hat{k}_o + (1 - \psi)\tilde{k}_o$ . Thus, there exists  $w_o > w_{o,\psi}$  and its corresponding  $x_o \in \Gamma(\hat{w}, b^-, z, s)$  that give a higher  $V$ , a contradiction. So,  $k_o(w, b^-, z, s)$  is unique for all  $w, b^-, z, s$ .

For  $w \geq \hat{w} \geq \bar{w}(b^-, z, s)$ , let  $\hat{x}_o$  attain  $V(\hat{w}, b^-, z, s)$  and  $x_o$  attain  $V(w, b^-, z, s)$ . Suppose  $\hat{k}_o \neq k_o$ . Then by concavity of  $\Gamma(w, b^-, z, s)$  in  $w$  [Lemma 1] and similar arguments as above, there exists a feasible choice such that  $V(w_\psi, b^-, z, s) > \psi V(w, b^-, z, s) + (1 - \psi)V(\hat{w}, b^-, z, s)$ , contradicting the fact that  $V$  is linear in  $w$  for  $w \geq \bar{w}(b^-, z, s)$ . Hence,  $k_o(w, b^-, z, s) = \bar{k}_o(b^-, z, s)$  for all  $w \geq \bar{w}(b^-, z, s)$ .

From above argument, I know that for  $w \geq \bar{w}$ , the firm can always choose the same optimal choices  $\bar{k}, \bar{b}$ , and  $\bar{w}$ .

Part (iii): Now, I will prove the uniqueness of optimal new debt level,  $b_o, b_{S,o}$ , end period worth net,  $w_o(z')$ , and dividend,  $d_o$ .

For any optimal policy  $x_o$ , if  $w_o(z') \geq \bar{w}(b, z', 0)$  for all  $z'$ , then by Proposition 4,  $w_o(z') \geq \bar{w}(b_o, z', s')$  and  $\mu(\hat{w}(z'), \hat{b}, z', s') = 1$  for all  $z'$ . In this case, from the optimality conditions, I know that either  $\lambda > 0$  or  $\gamma_s > 0$ . If  $\lambda > 0$ , then  $\gamma_l(w(z'), b_l, z', 0) > 0$  for some future state  $(z', 0)$ . Since both conditions (1.4) and (1.5) hold as equality,  $w_o(z')$  are uniquely determined by  $k_o$ . Since  $\gamma_l(w(z'), b, z', 0)$  is strictly decreasing in  $b$ ,  $b_o$  is also unique. Then,  $b_{S,o}$  and  $d_o$  are unique by equations (1.5) and (1.3). On the other hand, if  $\gamma_s > 0, \lambda = 0$  then  $\gamma_l > 0$  since  $\mu \geq 1 > \beta R_L$  and by optimality

condition (1.11). Thus,  $b_{S,o} = 0$  and  $b_o = b^-$ , so  $b_{S,o}$ ,  $b_o$  are unique. Hence,  $w(z')$  and  $d_o$  are unique by equations (1.4) and (1.3).

Now Suppose  $x$  and  $\hat{x}$  both obtain the optimal value and  $w(z') < \bar{w}(b, z', 0)$  and  $\hat{w}(z') < \bar{w}(\hat{b}, z', 0)$  for some state  $(z', 0)$ . I denote such state by  $\tilde{z}$ . Since equations (1.4) hold for all  $z'$ , if  $w(z') \neq \hat{w}(z')$  for some state  $z'$ , then  $w(z') \neq \hat{w}(z')$  for all state  $z'$  and vice versa. Suppose either  $w(z') \neq \hat{w}(z')$  for some  $z'$  or  $b \neq \hat{b}$ , then either  $w(\tilde{z}) \neq \hat{w}(\tilde{z})$  or  $b \neq \hat{b}$ . By Proposition 1, the region  $\{(w', b) : d(w', b) = 0\}$  is convex and the value function  $V(w', b)$  is strictly concave in that region. Hence, a convex combination between  $\{w(\tilde{z}), b\}$  and  $\{\hat{w}(\tilde{z}), \hat{b}\}$  is feasible and attains a strictly higher value, a contradiction. Therefore, it must be that  $b = \hat{b}$  and  $w(z') = \hat{w}(z')$  for all state  $z'$ . Thus,  $b_o$  and  $w_o(z')$  are unique if  $w_o(z') < \bar{w}(b_o, z', 0)$  for some  $(z', 0)$ . Finally,  $d_o$  and  $b_{S,o}$  are also unique by equations (1.4) and (1.3).  $\square$

*Proof of Proposition 4.* Figure 1.7 shows the value functions for two different levels of long-term debt. I use the figure to illustrate the relations between the points mentioned in the following proof.

By Proposition 1, for two levels of long-term debt,  $b_L^- > \tilde{b}_L^-$ , both  $V(w, b_L^-)$  and  $V(w, \tilde{b}_L^-)$  are increasing and concave in  $w$ . Also,  $V(w, b_L^-) \geq V(w, \tilde{b}_L^-)$  at any point  $w$  with strictly inequality when financing constraint is binding at  $w, b_L^-$ . Further, by part (i) of Proposition 2,  $V(w, b_L^-)$  and  $V(w, \tilde{b}_L^-)$  eventually stay parallel since  $\mu(w, b_L^-) = \mu(w, \tilde{b}_L^-) = 1$  for  $w \geq \max\{\bar{w}(\tilde{b}_L^-, z, 0), \bar{w}(b_L^-, z, 0)\}$ . Finally, as  $w \rightarrow 0$ , both  $V(w, b_L^-)$  and  $V(w, \tilde{b}_L^-) \rightarrow 0$ . Hence,  $\mu(w, b_L^-)$  falls to 1 before  $\mu(w, \tilde{b}_L^-)$  does. That is,  $\bar{w}(\tilde{b}_L^-, z, 0) > \bar{w}(b_L^-, z, 0)$ . Moreover, since  $V(w, \tilde{b}_L^-)$  is strictly concave before  $\bar{w}(\tilde{b}_L^-, z, 0)$ ,  $\exists \tilde{w}(\tilde{b}_L^-, b_L^-)$  s.t.  $\mu(w, \tilde{b}_L^-) > \mu(w, b_L^-)$  for all  $w \in (\tilde{w}(\tilde{b}_L^-, b_L^-), \bar{w}(\tilde{b}_L^-, z, 0))$ .  $\square$

## Appendix C: 2-Period Model Formulation, FOC, and SOC

*Proof of Lemma 7.* Here I first solve the problem backward starting from time 1.

At time 1, when  $s = 1$ , the firm is solving the following problem:

$$\max_{d_1, k_2, b_2^L, w_2(A_2)} d_1 + \beta E_1 w_2(A_2),$$

$$w_1 + b_2 + b_2^L \geq d_1 + k_2,$$

$$A_2 k_2 + (1 - \delta)k_2 - Rb_2 - R_L b_2^L \geq w_2(A_2)$$

$$\theta(1 - \delta)k_2 \geq Rb_2 + R_L b_2^L,$$

$$d_1 \geq 0, \quad b_2^L \geq 0, \quad b_2 \geq 0.$$

With linear production function under Assumption 3, the solution is:  $d_1 = 0$ ,  $k_2 = \frac{w_1}{q}$ ,  $b_2 = (\frac{1}{q} - 1)w_1$ ,  $b_2^L = 0$ ,  $w_2(A_2) = \frac{Ret_2}{q}w_1$ .

At time 1, when  $s = 0$ , the firm is solving the following problem:

$$\max_{d_1, k_2, b_2^L, w_2(A_2)} d_1 + \beta E_1 w_2(A_2),$$

$$w_1 + b_2 + b_2^L \geq d_1 + k_2,$$

$$A_2 k_2 + (1 - \delta)k_2 - Rb_2 - R_L b_2^L \geq w_2(A_2)$$

$$\theta(1 - \delta)k_2 \geq Rb_2 + R_L b_2^L,$$

$$b_1^L \geq b_2^L,$$

$$0 \geq b_2,$$

$$d_1 \geq 0, \quad b_2^L \geq 0, \quad b_2 \geq 0$$

As in the first case, the firm always wants to invest all in capital and payout nothing at time 1.

If  $b_1^L \geq (\frac{1}{q_L} - 1)w_1$ , then the firm is constrained by  $w_1$ , hence  $d_1 = 0$ ,  $k_2 = \frac{w_1}{q_L}$ ,  $b_2 = 0$ ,  $b_2^L = (\frac{1}{q_L} - 1)w_1$ ,  $w_2(A_2) = \frac{Ret_2}{q_L}w_1$ .

If  $b_1^L < (\frac{1}{q_L} - 1)w_1$ , then the firm is constrained by  $b_1^L$ , hence  $d_1 = 0$ ,  $k_2 = w_1 + b_1^L$ ,  $b_2^L = b_1^L$ ,  $b_2 = 0$ ,  $w_2(A_2) = \hat{R}et_2(w_1 + b_1^L) - R_L b_1^L$ .  $\square$

Assuming a continuum of productivity states, and let  $g(A_1)$  and  $\tilde{g}(A_2|A_1)$  be the marginal density of  $A_1$  and the conditional density of  $A_2$  given  $A_1$ , at time 0, the problem becomes:

$$\begin{aligned} \max_{d_0, k_1, b_1, b_1^L, w_1(A_1)} d_0 + \beta^2 & \left\{ (1 - \pi) \int_{A^L}^{A^H} \int_{A^L}^{A^H} \frac{Ret(A_2)}{p} w_1(A_1) \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \right. \\ & + \pi \int_{\underline{A}}^{\underline{A}} \int_{A^L}^{A^H} \frac{Ret(A_2)}{p_L} w_1(A_1) \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \\ & + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} \hat{R}et(A_2) w_1(A_1) \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \\ & \left. + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} [\hat{R}et(A_2) - R_L] \tilde{g}(A_2|A_1) b_1^L g(A_1) dA_2 dA_1 \right\} \quad (A.1) \end{aligned}$$

Subject to the budget constraints

$$w_0 + b_1 + b_1^L - k_1 \geq d_0,$$

$$A_1 k_1 + (1 - \delta) k_1 - R_L b_1^L - R b_1 \geq w_1(A_1),$$

the collateral constraint

$$\theta(1 - \delta) k_1 \geq R_L b_1^L + R b_1,$$

the non-negativity constraints

$$b_1 \geq 0, \quad b_1^L \geq 0, \quad d_0 \geq 0, \quad k_1 \geq 0, \quad w_1(A_1) \geq 0,$$

and where

$$\underline{A} \equiv \left\{ A_1 : w_1(A_1) = \frac{b_1^L p_L}{1 - p_L} \right\}.$$



Each line in the big curly bracket of equation A.1 represents a payoff for one of the three sets of states the firm may end up in at time 1. The first line is the payoff if the firm has external financing at time 1 ( $s = 1$ ). The second line is the payoff if the firm does not have external financing and does not have sufficient net worth to keep all existing long-term debt at time 1 ( $s = 0$ ,  $w_1(A_1) < \frac{b_1^L p_L}{1-p_L}$ ). The last line is the payoff if the firm does not have external financing but has sufficient net worth at time 1 ( $s = 0$ ,  $w_1(A_1) \geq \frac{b_1^L p_L}{1-p_L}$ ).

I consider the case in which the productivity shocks are profitable enough such that the firm will investment all in capital. Hence, the firm pays no dividends,  $d_0 = 0$ . Plus, the collateral constraint always binds.

Let  $\alpha$  be the proportion of net worth that is used to lever through long-term debt. More specifically,  $\alpha = \frac{p_L}{(1-p_L)} \frac{b_1^L}{w_0}$ . The interpretation is that if the firm uses  $\alpha w_0$  as the minimal downpayment to borrow with long-term debt, it will be able to raise  $b_1^L = \frac{(1-p_L)}{p_L} \alpha w_0$ .

Now the problem can be reformulated as follow: (since the whole problem is linear in  $w_0$ , I can divide it out and focus only on the proportional decision)

$$\begin{aligned}
V(\alpha) = \max_{\alpha} \quad & (1 - \pi) \int_{A^L}^{A^H} \int_{A^L}^{A^H} \frac{Ret(A_2)}{p} w_1(A_1) \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \\
& + \pi \int_{A^L}^{\underline{A}} \int_{A^L}^{A^H} \frac{Ret(A_2)}{p_L} w_1(A_1, \alpha) \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \\
& + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} \hat{Ret}(A_2) w_1(A_1, \alpha) \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \\
& + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - R_L] \tilde{g}(A_2|A_1) b_1^L(\alpha) g(A_1) dA_2 dA_1
\end{aligned}$$

where

$$w_1(A_1, \alpha) = Ret_1(A_1) Lev w_0 \quad (\text{A.2})$$

$$Ret_1(A_1) = A_1 + (1 - \delta)(1 - \theta) \quad (A.3)$$

$$Lev = \left[ \frac{\alpha}{p_L} + \frac{1 - \alpha}{p} \right] \quad (A.4)$$

$$b_1^L = \frac{(1 - p_L)}{p_L} \alpha w_0 \quad (A.5)$$

$$\underline{A} = \frac{\alpha}{Lev} - (1 - \delta)(1 - \theta) \quad (A.6)$$

$$p = 1 - R^{-1}\theta(1 - \delta), \quad p_L = 1 - R_L^{-1}\theta(1 - \delta). \quad (A.7)$$

The first order condition is given by:

$$\begin{aligned} & \left\{ (1 - \pi) \int_{A^L}^{A^H} \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p} \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1 \right. \\ & + \pi \int_{A^L}^{\underline{A}} \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1 \\ & + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} \left\{ \hat{Ret}(A_2) \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1 \left. \right\} \frac{(p_L - p)}{p_L p} \\ & = \pi \frac{1 - p_L}{p_L} \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - R_L] \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \end{aligned}$$

Notice that the terms involving derivatives of  $\underline{A}$  with respect to  $\alpha$  cancel out because at state  $\underline{A}$  the firm is indifferent between being constrained by net worth or being constrained by existing long-term debt. That is:

$$\begin{aligned} & -b_1^L(\alpha) \int_{A^L}^{A^H} [\hat{Ret}(A_2) - R_L] \tilde{g}(A_2|\underline{A}) g(\underline{A}) dA_2 \\ & + \left\{ \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A}) g(\underline{A}) \right\} dA_2 \right. \\ & - \left. \int_{A^L}^{A^H} \left\{ \hat{Ret}(A_2) \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A}) g(\underline{A}) \right\} dA_2 \right\} Lev(\alpha) = 0 \quad \forall \alpha \end{aligned}$$

The second order derivative is:

$$\begin{aligned}
\frac{\partial^2 V(\alpha)}{\partial \alpha^2} &= -\pi \frac{1-p_L}{p_L} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - R_L] \tilde{g}(A_2|\underline{A}) g(\underline{A}) dA_2 \\
&\quad - \left\{ \pi \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A}) g(\underline{A}) \right\} dA_2 \right. \\
&\quad \left. - \pi \int_{A^L}^{A^H} \left\{ \hat{Ret}(A_2) \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A}) g(\underline{A}) \right\} dA_2 \right\} \frac{(p_L - p)}{p_L p}
\end{aligned}$$

Which have the same sign as:

$$\begin{aligned}
& -\frac{1-p_L}{p_L} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - R_L] \tilde{g}(A_2|\underline{A}) dA_2 - \left\{ \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A}) \right\} dA_2 \right. \\
& \quad \left. - \int_{A^L}^{A^H} \left\{ \hat{Ret}(A_2) \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A}) \right\} dA_2 \right\} \frac{(p_L - p)}{p_L p} \\
&= \int_{A^L}^{A^H} \left\{ \left[ \hat{Ret}(A_2) - \frac{Ret(A_2)}{p_L} \right] Ret_1(\underline{A}) \frac{(p_L - p)}{p_L p} - [\hat{Ret}(A_2) - R_L] \frac{1-p_L}{p_L} \right\} \tilde{g}(A_2|\underline{A}) dA_2 \\
&= - \int_{A^L}^{A^H} \left\{ \left[ \frac{Ret(A_2)}{p_L} - \hat{Ret}(A_2) \right] Ret_1(\underline{A}) \frac{1}{p_L} \right\} \tilde{g}(A_2|\underline{A}) dA_2 < 0
\end{aligned}$$

Hence,  $\frac{\partial^2 V(\alpha)}{\partial \alpha^2} < 0 \quad \forall \alpha$ . Therefore, there is a unique  $\alpha$  for the problem.

## Appendix D: 2-Period Model comparative statics

*Proof of proposition 19.* Now suppose that  $A_2 = \rho A_1 + \epsilon$ . The density for  $A_1$  is  $g(A_1)$ .

The density for  $\epsilon$  is  $f(\epsilon)$  which is independent of  $A_1$ . Hence, the FOC is:

$$\begin{aligned}
& \left\{ (1 - \pi) \int_{A^L} \int^{A^H} \left\{ \frac{Ret(\rho A_1 + \epsilon)}{p} f(\epsilon) Ret(A_1) g(A_1) \right\} d\epsilon dA_1 \right. \\
& \quad + \pi \int_{A^L} \int^{\underline{A}} \left\{ \frac{Ret(\rho A_1 + \epsilon)}{p_L} f(\epsilon) Ret(A_1) g(A_1) \right\} d\epsilon dA_1 \\
& \quad \left. + \pi \int_{\underline{A}}^{A^H} \int \left\{ \hat{Ret}(\rho A_1 + \epsilon) f(\epsilon) Ret(A_1) g(A_1) \right\} d\epsilon dA_1 \right\} \frac{(p_L - p)}{p_L p} \\
& = \pi \frac{1 - p_L}{p_L} \int_{\underline{A}}^{A^H} \int [\hat{Ret}(\rho A_1 + \epsilon) - R_L] f(\epsilon) g(A_1) d\epsilon dA_1
\end{aligned}$$

or equivalently,

$$\begin{aligned}
& \left\{ (1 - \pi) \int_{A^L}^{A^H} \frac{Ret(\rho A_1)}{p} Ret(A_1) g(A_1) dA_1 + \pi \int_{A^L}^{\underline{A}} \frac{Ret(\rho A_1)}{p_L} Ret(A_1) g(A_1) dA_1 \right. \\
& \quad \left. + \pi \int_{\underline{A}}^{A^H} \hat{Ret}(\rho A_1) Ret(A_1) g(A_1) dA_1 \right\} \frac{(p_L - p)}{p_L p} \\
& = \pi \frac{1 - p_L}{p_L} \int_{\underline{A}}^{A^H} [\hat{Ret}(\rho A_1) - R_L] g(A_1) dA_1
\end{aligned}$$

And  $\frac{\partial^2 V(\alpha)}{\partial \alpha \partial \rho}$  becomes:

$$\begin{aligned}
& - \left\{ (1 - \pi) \int_{A^L}^{A^H} \frac{A_1}{p} Ret(A_1) g(A_1) dA_1 + \pi \int_{A^L}^{\underline{A}} \frac{A_1}{p_L} Ret(A_1) g(A_1) dA_1 \right. \\
& \quad \left. + \pi \int_{\underline{A}}^{A^H} A_1 Ret(A_1) g(A_1) dA_1 \right\} \frac{(p_L - p)}{p_L p} + \pi \frac{1 - p_L}{p_L} \int_{\underline{A}}^{A^H} A_1 g(A_1) dA_1
\end{aligned}$$

From the FOC, we have that

$$\begin{aligned}
& \frac{\partial^2 V(\alpha)}{\partial \alpha \partial \rho} \rho - \left\{ (1 - \pi) \int_{A^L}^{A^H} \frac{(1 - \delta)(1 - \theta)}{p} Ret(A_1) g(A_1) dA_1 \right. \\
& \quad + \pi \int_{A^L}^{\underline{A}} \frac{(1 - \delta)(1 - \theta)}{p_L} Ret(A_1) g(A_1) dA_1 \\
& \quad \left. + \pi \int_{\underline{A}}^{A^H} (1 - \delta) Ret(A_1) g(A_1) dA_1 \right\} \frac{(p_L - p)}{p_L p} \\
& \quad + \pi \frac{1 - p_L}{p_L} [(1 - \delta) - R_L] \int_{\underline{A}}^{A^H} g(A_1) dA_1 = 0
\end{aligned}$$

Since  $(1 - \delta) - R_L < 0$  and the term in curly bracket is positive,  $\frac{\partial^2 V(\alpha)}{\partial \alpha \partial \rho} > 0$ .

Therefore, by the SOC and implicit function theorem,  $\frac{\partial \alpha}{\partial \rho} > 0$ .  $\square$

*Proof of proposition 10.* Suppose for now the shocks are independent. Let  $\hat{Ret}_2 = E[\hat{Ret}(A_2)]$ ,  $Ret_2 = E[Ret(A_2)]$ , and  $Ret_1 = E[Ret(A_1)]$

The now FOC now becomes:

$$\begin{aligned}
& - \left\{ (1 - \pi) \frac{Ret_2}{q} Ret_1 + \pi \frac{Ret_2}{q_L} \int_{A^L}^{\underline{A}} Ret(A_1) g(A_1) dA_1 \right. \\
& \quad \left. + \pi \hat{Ret}_2 \int_{\underline{A}}^{A^H} Ret(A_1) g(A_1) dA_1 \right\} \frac{(p_L - p)}{p_L p} \\
& \quad + \Pr(A_1 > \underline{A}) \pi [\hat{Ret}_2 - R_L] \frac{1 - q_L}{q_L} = 0
\end{aligned}$$

And we have:

$$\begin{aligned}
\frac{\partial^2 V(\alpha)}{\partial \alpha \partial (EA_2)} = & - \left\{ (1 - \pi) \frac{Ret_1}{q} + \frac{\pi}{q_L} \int_{A^L}^{\underline{A}} Ret(A_1) g(A_1) dA_1 \right. \\
& \left. + \pi \int_{\underline{A}}^{A^H} Ret(A_1) g(A_1) dA_1 \right\} \frac{(p_L - p)}{p_L p} + \Pr(A_1 > \underline{A}) \pi \frac{1 - q_L}{q_L}
\end{aligned}$$

By manipulating the term, we obtain the following equation:

$$\begin{aligned}
& - \left\{ (1 - \pi) \frac{Ret_1}{q} [Ret_2 - \hat{Ret}_2 + R_L] + \frac{\pi}{q_L} [Ret_2 - \hat{Ret}_2 + R_L] \int_{A^L}^{\underline{A}} Ret(A_1) g(A_1) dA_1 \right. \\
& \left. + \pi R_L \int_{\underline{A}}^{A^H} Ret(A_1) g(A_1) dA_1 \right\} \frac{(p_L - p)}{p_L p} + \frac{\partial^2 V(\underline{A})}{\partial \underline{A} \partial (E_1 A_2)} [\hat{Ret}_2 - R_L] = \frac{\partial V(\alpha)}{\partial \alpha} = 0
\end{aligned}$$

Since  $[Ret_2 - \hat{Ret}_2 + R_L] > 0$ , the term in the curly bracket is positive. Also, since  $\hat{Ret}_2 - R_L > 0$ , we have  $\frac{\partial^2 V(\alpha)}{\partial \alpha \partial (E A_2)} > 0$ . Finally, by the second order condition and the implicit function theorem, we have:  $\frac{\partial \alpha}{\partial (E A_2)} > 0$ .  $\square$

*Proof of proposition 11.* Now suppose instead that  $A_1 + \epsilon$  is the productivity at time 1, and still  $g(A_1)$  is the marginal pdf for  $A_1$  and  $\tilde{g}(A_2|A_1)$  is the conditional pdf for  $A_2$  given  $A_1$ .

We have that  $\underline{A} = \frac{\alpha}{Lev} - (1 - \delta)(1 - \theta) - \epsilon$ , and the return for time 1 is now  $Ret(A_1) = A_1 + \epsilon + (1 - \delta)(1 - \theta)$ . Hence, the now FOC is:

$$\begin{aligned}
& \left\{ (1 - \pi) \int_{A^L}^{A^H} \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p} \tilde{g}(A_2|A_1) Ret_1(A_1 + \epsilon) g(A_1) \right\} dA_2 dA_1 \right. \\
& + \pi \int_{A^L}^{\underline{A}} \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|A_1) Ret_1(A_1 + \epsilon) g(A_1) \right\} dA_2 dA_1 \\
& \left. + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} \left\{ \hat{Ret}(A_2) \tilde{g}(A_2|A_1) Ret_1(A_1 + \epsilon) g(A_1) \right\} dA_2 dA_1 \right\} \frac{(p_L - p)}{p_L p} \\
& = \pi \frac{1 - p_L}{p_L} \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - R_L] \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1
\end{aligned}$$

where  $\underline{A} = \frac{\alpha}{Lev} - (1 - \delta)(1 - \theta) - \epsilon$ . So  $\frac{\partial^2 V(\alpha)}{\partial \alpha \partial \epsilon}$  becomes:

$$\begin{aligned}
& \pi \left\{ \left[ \int_{A^L}^{A^H} \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A} + \epsilon) g(\underline{A}) dA_2 \right. \right. \\
& \quad - \int_{A^L}^{A^H} \hat{Ret}(A_2) \tilde{g}(A_2|\underline{A}) Ret_1(\underline{A} + \epsilon) g(\underline{A}) dA_2 \left. \right] \frac{(p_L - p)}{p_L p} \\
& \quad - \frac{1 - p_L}{p_L} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - R_L] \tilde{g}(A_2|\underline{A}) g(\underline{A}) dA_2 \left. \right\} \\
& - \left\{ (1 - \pi) \int_{A^L}^{A^H} \int_{A^L}^{A^H} \frac{Ret(A_2)}{p} \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \right. \\
& \quad + \pi \int_{A^L}^{\underline{A}} \int_{A^L}^{A^H} \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \\
& \quad \left. + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} \hat{Ret}(A_2) \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1 \right\} \frac{(p_L - p)}{p_L p}
\end{aligned}$$

The three terms together in the first curly bracket are the increase in marginal benefit of long-term debt due to the increase in time 1 productivity. The three terms together in the second curly bracket are the increase in marginal cost of long-term debt due to the increase in time 1 productivity. As long as  $\pi$  is small, the sum in the second bracket dominates that in the first bracket. Hence, for  $\pi$  small,  $\frac{\partial^2 V(\alpha)}{\partial \alpha \partial \epsilon} < 0$ . Thus by implicit function theorem and SOC,  $\frac{\partial \alpha}{\partial \epsilon} < 0$ .  $\square$

*Proof of proposition 12.* Now suppose that the interest rates for short-term and long-term debt are  $\lambda R$  and  $\lambda R_L$  where  $\lambda$  is a scalar which represents a proportional increase or decrease in the interest rates.  $p_L = 1 - (\lambda R_L)^{-1} \theta (1 - \delta)$  and  $p = 1 - (\lambda R)^{-1} \theta (1 - \delta)$ . Then the FOC is:

$$\begin{aligned}
& \left\{ (1 - \pi) \int_{A^L}^{A^H} \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p} \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1 \right. \\
& + \pi \int_{A^L}^{\underline{A}} \int_{A^L}^{A^H} \left\{ \frac{Ret(A_2)}{p_L} \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1 \\
& \left. + \pi \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} \left\{ \hat{Ret}(A_2) \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1 \right\} \frac{(p_L - p)}{p_L p} \\
& = \pi \frac{1 - p_L}{p_L} \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - \lambda R_L] \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1
\end{aligned}$$

For convenience, let's denote the sum of the three terms in the first curly brackets by  $E[Ret_2 \times Ret_1; \underline{A}]$ . Also, let's define:

$$E[Ret(A_2); A_1 < \underline{A}] = \int_{A^L}^{\underline{A}} \int_{A^L}^{A^H} \left\{ Ret(A_2) \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1,$$

$$E[Ret(A_2)] = \int_{A^L}^{\underline{A}} \int_{A^L}^{A^H} \left\{ Ret(A_2) \tilde{g}(A_2|A_1) Ret_1(A_1) g(A_1) \right\} dA_2 dA_1,$$

$$\text{and } E[\hat{Ret}(A_2) - R_L; A_1 > \underline{A}] = \int_{\underline{A}}^{A^H} \int_{A^L}^{A^H} [\hat{Ret}(A_2) - \lambda R_L] \tilde{g}(A_2|A_1) g(A_1) dA_2 dA_1.$$

Now take the derivative of  $\frac{V(\alpha)}{\partial \alpha}$  with respect to  $\lambda$ , we obtain:

$$\begin{aligned}
\frac{\partial^2 V(\alpha)}{\partial \alpha \partial \lambda} &= E[Ret_2 \times Ret_1; \underline{A}] \left[ \frac{1}{p^2} \frac{\theta(1 - \delta)}{(\lambda^2 R)} - \frac{1}{p_L^2} \frac{\theta(1 - \delta)}{(\lambda^2 R_L)} \right] \\
&- \left\{ \frac{(1 - \pi) \theta(1 - \delta)}{p^2 (\lambda^2 R)} E[Ret(A_2)] + \frac{\pi \theta(1 - \delta)}{p_L^2 (\lambda^2 R_L)} E[Ret(A_2); A_1 < \underline{A}] \right\} \frac{(p - p_L)}{p_L p} \\
&- \pi \frac{1 - p_L}{p_L} \Pr[A_1 > \underline{A}] R_L - \frac{\pi \theta(1 - \delta)}{p_L^2 (\lambda^2 R_L)} E[\hat{Ret}(A_2) - R_L; A_1 > \underline{A}]
\end{aligned}$$

Now evaluate the above derivative at  $\lambda = 1$ ,



$$\begin{aligned}
& E[Ret_2 \times Ret_1; \underline{A}] \left[ \frac{1}{p^2} \frac{\theta(1-\delta)}{R} - \frac{1}{p_L^2} \frac{\theta(1-\delta)}{R_L} \right] \\
& - \left\{ \frac{(1-\pi)}{p^2} \frac{\theta(1-\delta)}{R} E[Ret(A_2)] + \frac{\pi}{p_L^2} \frac{\theta(1-\delta)}{R_L} E[Ret(A_2); A_1 < \underline{A}] \right\} \frac{(p-p_L)}{p_L p} \\
& - \pi \frac{1-p_L}{p_L} \Pr[A_1 > \underline{A}] R_L - \frac{\pi}{p_L^2} \frac{\theta(1-\delta)}{R_L} E[\hat{Ret}(A_2) - R_L; A_1 > \underline{A}] \\
& < E[Ret_2 \times Ret_1; \underline{A}] \left( \frac{1}{p} - \frac{1}{p_L} \right) \left( \frac{1}{p} + \frac{1}{p_L} \right) \frac{\theta(1-\delta)}{R_L} \\
& - \pi \left( \frac{1}{p_L} - 1 \right) \left( \frac{1}{p} + \frac{1}{p_L} \right) \frac{\theta(1-\delta)}{R_L} E[\hat{Ret}(A_2) - R_L; A_1 > \underline{A}] - \pi \frac{1-p_L}{p_L} \Pr[A_1 > \underline{A}] R_L \\
& - \left\{ \frac{(1-\pi)}{p^2} \frac{\theta(1-\delta)}{R} E[Ret(A_2)] + \frac{\pi}{p_L^2} \frac{\theta(1-\delta)}{R_L} E[Ret(A_2); A_1 < \underline{A}] \right\} \frac{(p-p_L)}{p_L p} \\
& < \left\{ \frac{(1-\pi)}{p} E[Ret(A_2)] + \frac{\pi}{p_L} E[Ret(A_2); A_1 < \underline{A}] \right\} \left( \frac{1}{p} - \frac{1}{p_L} \right) \frac{\theta(1-\delta)}{R p} \\
& - \pi \frac{1-p_L}{p_L} \Pr[A_1 > \underline{A}] R_L
\end{aligned}$$

Hence, if  $\frac{\theta(1-\delta)}{Rp}$  is small or if  $\pi$  is big, then  $\frac{\partial^2 V(\alpha)}{\partial \alpha \partial \lambda} < 0$ . Therefore, by implicit function theorem and SOC,  $\frac{\partial \alpha}{\partial \lambda} < 0$ . Here  $\frac{\theta(1-\delta)}{R}$  is the proportion per each unit of capital that can be borrowed using short-term debt.  $\frac{1}{p}$  is the leverage from short-term financing. Hence, when the firm is constrained (i.e. when  $\frac{\theta(1-\delta)}{Rp}$  is small), it uses even more short-term debt when it becomes more constrained (i.e. when  $\lambda$  increases) .

Note: When  $\frac{\theta(1-\delta)}{R}$  is low,  $\frac{1}{p}$  is also low. So, the first inequality is due to the fact that  $(\frac{1}{p_L} - 1)(\frac{1}{p} + \frac{1}{p_L}) = \frac{1}{p_L^2} + [\frac{1}{p_L} \frac{1}{p} - (\frac{1}{p} + \frac{1}{p_L})]$  which is well below  $\frac{1}{p_L^2}$  when  $\frac{1}{p}$  and  $\frac{1}{p_L}$  are low (i.g. when  $\frac{1}{p} < 2$ ). From the first order condition,  $E[Ret_2 \times Ret_1; \underline{A}](\frac{1}{p} - \frac{1}{p_L}) - \pi(\frac{1}{p_L} - 1)E[\hat{Ret}(A_2) - R_L; A_1 > \underline{A}] = 0$ . Hence, the second inequality is due to the first order condition and the fact that  $\frac{1}{Rp} > \frac{1}{R_L p_L}$ .  $\square$

## Appendix E: Resource Allocation and Debt Maturity

First I prove the following lemma that will be used in proofs later on.

**Lemma 22.** *Suppose that  $\int_0^\infty f(x) = 0$ ,  $f(x) \leq 0$  for  $x \leq x_0$  and  $f(x) > 0$  for  $x > x_0$ . Let  $g(x) \geq 0$  be a monotonic function defined on  $[0, \infty)$ . Then  $\int_0^\infty f(x)g(x) \geq 0$ , if  $g(x)$  is increasing, and  $\int_0^\infty f(x)g(x) \leq 0$ , if  $g(x)$  is decreasing. The strict inequality holds if  $g(x)$  is strictly increasing or decreasing.*

**Proof.** I prove the case when  $g(x)$  is increasing. The same proof applies if  $g(x)$  is decreasing.

$$\begin{aligned} \int_0^\infty f(x)g(x)dx &= \int_0^{x_0} f(x)g(x)dx + \int_{x_0}^\infty f(x)g(x)dx \\ &\geq \int_0^{x_0} f(x)g(x_0)dx + \int_{x_0}^\infty f(x)g(x_0)dx \\ &= g(x_0) \int_0^\infty f(x)dx = 0. \end{aligned}$$

The inequality holds because  $f(x) \leq 0$  for  $x \leq x_0$ ,  $f(x) > 0$  for  $x > x_0$ , and because  $g(x) \leq g(x_0)$  for all  $x \leq x_0$ ,  $g(x_0) \leq g(x)$  for all  $x_0 \leq x$ . The inequality becomes strict if  $g(x) < g(x_0)$  for all  $x < x_0$ ,  $g(x_0) < g(x)$  for all  $x_0 < x$ .

Using the optimality conditions and plug in the expression for  $c$  and  $c'$  for each financial state  $s$  I have the following equations for the unconstrained problem (when  $0 \leq a' + b$  is not binding)

$$\begin{aligned}
\frac{1}{A(z, 1)a - a'} &= \beta \int_0^\infty \left( \frac{pA(z', 0)}{A(z, 0)(a' + b) - R_L b - a''} \right. \\
&\quad \left. + \frac{(1-p)A(z', 1)}{A(z', 1)a' - a''} \right) \psi(z'|z) dz', \quad s = 1, \\
\frac{1}{A(z, 0)(a + b^-) - R_L b^- - a'} &= \beta \int_0^\infty \left( \frac{pA(z', 0)}{A(z, 0)(a' + b) - R_L b - a''} \right. \\
&\quad \left. + \frac{(1-p)A(z', 1)}{A(z', 1)a' - a''} \right) \psi(z'|z, \sigma') dz', \quad s = 0, \\
\int_0^\infty \frac{A(z', 0) - R_L}{A(z, 0)(a' + b) - R_L b - a''} \psi(z'|z) dz' &= 0, \quad \text{for } s = 1.
\end{aligned}$$

where the returns on saving for states  $s = 1, 2$  as  $A(z, 0) = z\pi + 1 - \delta$  and  $A(z, 1) = \lambda \max\{z\pi - r - \delta, 0\} + R$  respectively.

Suppose that all optimal decisions are linear in wealth I write the choice variables as:

$$\begin{aligned}
c &= (1 - \phi_a^1)A(z, 1)a & b &= \phi_b(z)A(z, 1)a & a' &= \phi_a^1 A(z, 1)a & \text{for } s = 1 \\
c &= (1 - \phi_a^0)\{A(z, 0)(a + b^-) - R_L b^-\} & a' &= \phi_a^0\{A(z, 0)(a + b^-) - R_L b^-\} & \text{for } s = 0
\end{aligned}$$

**Now I prove that the current period maximization problem is concave in  $\phi_a^1$ ,  $\phi_a^0$ , and  $\phi_b$ .**

**Proof:**

First note that  $V(a', b, z', s)$  is concave in  $a'$  and  $b$ . Since  $\phi_a^1$ ,  $\phi_a^0$ , and  $\phi_b$  are linearly related to  $a'$  and  $b$ ,  $V(a', b, z', s)$  is also concave in  $\phi_a^1$ ,  $\phi_a^0$ , and  $\phi_b$ . Moreover, the current utility function  $\log(\text{wealth} - a')$  is concave in  $a'$ . Along with linear constraints, I conclude that the objective function is concave in  $\phi_a^1$ ,  $\phi_a^0$ , and  $\phi_b$ . Since I have proved

that the current objective function is concave in  $\phi_a^1$ ,  $\phi_a^0$ , and  $\phi_b$ , I can find the solution by the following procedure.

In first step, I guess and verify the solution for the unconstrained problem. Then I prove monotonicity of the policy function  $\phi_b(z)$  over  $R_L$  and  $z$ . Finally, I derive conditions under which constraint (2.3) is binding and characterize the solution for the general case.

The solution for the unconstrained problem is determined by the following necessary and sufficient optimality conditions:

$$\frac{1}{1 - \phi_a^0(z)} = \beta \int_0^\infty \left( \frac{p}{\phi_a^0(z)(1 - \phi_a^0(z'))} + \frac{1 - p}{\phi_a^0(z)(1 - \phi_a^1(z'))} \right) \psi(z'|z) dz' \quad (\text{A.8})$$

$$\frac{1}{1 - \phi_a^1(z)} = \beta \int_0^\infty \left( \frac{p(z'\pi + 1 - \delta)}{[(\phi_a^1(z) + \phi_b(z))(z'\pi + 1 - \delta) - R_L \phi_b(z)](1 - \phi_a^0(z'))} \right. \quad (\text{A.9})$$

$$\left. + \frac{1 - p}{\phi_a^1(z)(1 - \phi_a^1(z'))} \right) \psi(z'|z) dz' \quad (\text{A.10})$$

$$\int_0^\infty \frac{z'\pi + 1 - \delta - R_L}{[(\phi_a^1(z) + \phi_b(z))(z'\pi + 1 - \delta) - R_L \phi_b(z)](1 - \phi_a^0(z'))} \psi(z'|z) dz' = 0. \quad (\text{A.11})$$

Suppose  $\phi_a^0(z) = \phi_a^1(z) = \beta$  and  $\phi_b(z) > -\beta$  after simplification, equations (A.9) and (A.11) become:

$$\frac{1}{\beta} = \frac{1}{\beta + \phi_b(z)} + \frac{\phi_b(z)}{\beta + \phi_b(z)} \int_0^\infty \frac{R_L \psi(z'|z)}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L \phi_b(z)]} dz'.$$

$$\frac{1}{\beta} = \int_0^\infty \frac{R_L \psi(z'|z)}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L \phi_b(z)]} dz'.$$

which can both be satisfied if  $\phi_b(z)$  is the solution to  $\phi_b(z) > -\beta$  and  $\frac{1}{\beta} = \int_0^\infty \frac{R_L \psi(z'|z)}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L \phi_b(z)]} dz'$ .

Hence, a solution to the system of equations is  $\phi_a^0(z) = \phi_a^1(z) = \beta$ , and  $\phi_b(z)$  is defined by:  $\int_0^\infty \frac{R_L \psi(z'|z)}{(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L \phi_b} dz' = \frac{1}{\beta}$  and  $\phi_b(z) > -\beta$ .

**Now I prove that  $\phi_b(z)$  is decreasing in  $R$  for any  $z$ . That is for  $\phi_b > -\beta$ ,**

**$\frac{\partial \phi_b}{\partial R_L} < 0$ . Proof:**

By implicit function theorem, I have:

$$\frac{\partial \phi_b}{\partial R_L} = \frac{\int_0^\infty \frac{(\beta + \phi_b)(z'\pi + 1 - \delta)\psi(z')}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]^2} dz'}{\int_0^\infty \frac{R[z'\pi + 1 - \delta - R]\psi(z')}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]^2} dz'}.$$

First, the numerator is always positive for  $\phi_b \geq -\beta$ .

Now notice that  $\int_0^\infty \frac{\psi(z')[z'\pi + 1 - \delta - R_L]}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]} dz' = 0$  by equation (A.11) from the first order conditions. Plus,  $\frac{1}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]}$  is decreasing in  $z'$  for  $\phi_b > -\beta$ . Thus, by lemma 22 I have  $\int_0^\infty \frac{\psi(z')[z'\pi + 1 - \delta - R_L]}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]^2} dz' < 0$ , for  $\phi_b > -\beta$ . Therefore,  $\frac{\partial \phi_b}{\partial R_L} < 0$  for  $\phi_b > -\beta$ .

Proof for  $\frac{1}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]}$  is decreasing in  $z'$  for  $\phi_b > -\beta$ .

$$\frac{\partial \frac{1}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]}}{\partial z'} = \frac{-(\beta + \phi_b)\pi}{[(\beta + \phi_b)(z'\pi + 1 - \delta) - R_L \phi_b]^2} < 0 \text{ for } \phi_b > -\beta.$$

**Now I prove that  $\phi_b(z)$  is increasing in  $z$  if  $\psi(z'|z + \varepsilon)$  F.O.S.D.  $\psi(z'|z)$  (or  $z$  is positively autocorrelated).**

**Proof of  $\frac{\partial \phi_b}{\partial z} < 0$  for  $\phi_b > -\beta$ :**

Since  $\phi_a^1$  is independent of  $z$ , I obtain the following by differentiating both sides of equation (A.11) with respect to  $z$ :

$$\begin{aligned}
& \int_0^\infty \left( \frac{z'\pi + 1 - \delta - R_L}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)]} \right) \frac{\partial \psi(z'|z)}{\partial z} dz' \\
&= \tilde{\phi}_b(z) \int_0^\infty \left( \frac{(z'\pi + 1 - \delta - R_L)}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)]} \right. \\
&\quad \times \left. \frac{(z'\pi + 1 - \delta - R_L)}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)]} \right) \psi(z'|z) dz'
\end{aligned}$$

Since  $\frac{\partial \frac{z'\pi + 1 - \delta - R_L}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)]}}{\partial z'} = \frac{\beta R_L \pi}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)]^2} > 0$ , Hence, the right hand side is positive because  $\psi(z'|z + \varepsilon)$  F.O.S.D.  $\psi(z'|z)$ .

Also since  $\int_0^\infty \frac{\psi(z') [z'\pi + 1 - \delta - R_L]}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)]} dz' = 0$  by equation (A.11) and since  $\frac{z'\pi + 1 - \delta - R_L}{[(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)]}$  is increasing in  $z'$ , by lemma 22, the integral on the left hand side is positive. Therefore,  $\tilde{\phi}_b(z) > 0$ .

Therefore, the optimal proportional long-term debt decision,  $\phi_b$ , is increasing in  $z$  and decreasing in  $R_L$ ,

By equation (A.11) above, I obtain that when for the lowest level of productivity  $z_L$ ,  $E(z'|z_L)\pi + 1 - \delta > R_L$ , then the constraint  $0 \leq a' + b$  is always slack for all  $z$ , so the above solution holds.

Also, I know that for a given  $R_L$ ,  $\phi_b$  monotonically decreases over  $z$ . Hence, for  $z > \underline{z}$ ,  $\phi_b$  is defined by  $\int_0^\infty \frac{R_L \psi(z'|z)}{(\beta + \phi_b(z))(z'\pi + 1 - \delta) - R_L\phi_b(z)} dz' = \frac{1}{\beta}$ , and for  $z < \underline{z}$ ,  $\phi_b = -\beta$ , where  $\underline{z}$  satisfies  $R_L = E(z'|\underline{z})\pi + 1 - \delta$ .

## Proof for Proposition 19

Suppose the productivity process is:  $\log(z') = \rho \log(z) - \frac{\sigma^2}{2} + \sigma \varepsilon$ , where  $\varepsilon \sim N(0, 1)$ . Then  $z' = z^\rho e^{\sigma \varepsilon - \frac{\sigma^2}{2}}$  and the FOCs in terms of  $z$  and  $\varepsilon$  are:

$$\frac{\partial \phi_b}{\partial \rho} = \frac{-\int_{-\infty}^{\infty} \frac{z^\rho e^{\sigma\varepsilon - \frac{\sigma^2}{2}} R_L \log(z) \pi f(\varepsilon)}{[(\beta + \phi_b)(z^\rho e^{\sigma\varepsilon - \frac{\sigma^2}{2}} \pi + 1 - \delta) - R\phi_b]^2} d\varepsilon}{\int_{-\infty}^{\infty} \frac{R_L [z^\rho e^{\sigma\varepsilon - \frac{\sigma^2}{2}} \pi + 1 - \delta - R_L] f(\varepsilon)}{[(\beta + \phi_b)(z^\rho e^{\sigma\varepsilon - \frac{\sigma^2}{2}} \pi + 1 - \delta) - R\phi_b]^2} d\varepsilon}.$$

Since  $\int_{-\infty}^{\infty} \frac{[z^\rho e^{\sigma\varepsilon - \frac{\sigma^2}{2}} \pi + 1 - \delta - R_L] f(\varepsilon)}{[(\beta + \phi_b)(z^\rho e^{\sigma\varepsilon - \frac{\sigma^2}{2}} \pi + 1 - \delta) - R_L \phi_b]} d\varepsilon = 0$  by equation (A.11) and since

$\frac{R_L}{[(\beta + \phi_b(z))(z' \pi + 1 - \delta) - R_L \phi_b(z)]}$  is decreasing in  $z'$ , by lemma 22, the denominator is always negative. The numerator is positive if  $\log(z) < 1$  and negative if  $\log(z) > 1$ . Therefore,  $\frac{\partial \phi_b}{\partial \rho} < 0$ , if  $\log(z) < 1$ , and  $\frac{\partial \phi_b}{\partial \rho} > 0$ , if  $\log(z) > 1$ .

# Bibliography

- Acharya, Viral, Douglas Gale, and Tanju Yorulmazer, 2011, Rollover Risk and Market Freezes, *Journal of Finance*, 66, 1177-1209.
- Barclay, Michael, and Clifford Smith Jr., 1995, The Maturity Structure of Corporate Debt, *Journal of Finance*, 50, 609-631.
- Barclay, Michael, Leslie Marx, and Clifford Smith, 2003, The joint determination of leverage and maturity, *Journal of Corporate Finance*, 9, 149-167.
- Barnea, Amir, Robert Haugen, and Lemma Senbet, 1980, A Rational for Debt Maturity Structure and Call Provisions in the Agency Theoretic Framework, *Journal of Finance*, 35, 1223-1234.
- Benmelech, Efraim, 2006, Managerial entrenchment and debt maturity: Theory and evidence, working paper, Department of Economics, Harvard University.
- Benmelech, Efraim, 2008, Asset salability and debt maturity: evidence from nineteenth-century american railroads, *Review of Financial Studies*, 22, 1545-1583.
- Benmelech, Efraim, Mark Garnaize, and Tobias Moskowitz, 2005, Do Liquidation Values Affect Financial Contracts? Evidence from Commercial Zoning Laws, *Quarterly Journal of Economics*, 120, 1121-1154.
- Berglof, Eric, and Ernst-Ludwig von Thadden, 1994, Short-Term versus Long-Term Interests: Capital Structure with Multiple Investors, *Quarterly Journal of Economics*, 109, 1055-1084.
- Bharath, Sreedhar, and Tyler Shumway, 2008, Forecasting default with the Merton distance to default model, *Review of Financial Studies*, 21, 1339-1369.
- Bloom, Nicholas, 2009, The Impact of Uncertainty Shocks, *Econometrica*, 77, 623-685.
- Bloom, Nicholas, Stephen Bond, and John Van Reenen, 2007, Uncertainty and Investment Dynamics, *Review of Economic Studies*, 74, 391-415.



- Broner, Fernando, Guido Lorenzoni, and Sergio Schmukler, 2013, Why do emerging economies borrow short term?, *Journal of the European Economic Association*, 11, 67-100.
- Brunnermeier, Markus and Martin Oehmke, 2013, The Maturity Rat Race, *Journal of Finance*, 68, 483-521.
- Calomiris, Charles, and Charles Kahn, 1991, The role of demandable debt in structuring optimal banking arrangements, *American Economic Review*, 81, 497-513.
- Cheng, Ing-Haw, and Konstantin Milbradt, 2012, The Hazards of Debt: Rollover Freezes Incentives, and Bailouts, *Review of Financial Studies*, 25, 1070-1110.
- Childs, Paul, David Mauer, and Steven Ott, 2005, Interactions of corporate financing and investment decisions: The effects of agency conflicts, *Journal of Financial Economics*, 76, 667-690.
- Datta, Sudip, Mai Iskandar-Datta, and Kartik Raman, 2005, Managerial Stock Ownership and the Maturity Structure of Corporate Debt, *Journal of Finance*, 60, 2333-2350.
- Diamond, Douglas, 1991, Debt maturity structure and liquidity risk, *Quarterly Journal of Economics*, 106, 709-737.
- Diamond, Douglas, and Zhiguo He, 2014, A Theory of Debt Maturity: The Long and Short of Debt Overhang, *Journal of Finance*, 69, 719-762.
- Diamond, Douglas, and Raghuram Rajan, 2001, Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking, *Journal of Political Economy*, 109, 287-327.
- Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, 2005, Over-the-Counter Markets, *Econometrica*, 73, 1815-1847.
- Eisenbach, Thomas, 2013, Rollover Risk as Market Discipline: A Two-Sided Inefficiency, working paper, Federal Reserve Bank of New York.
- Eisfeldt, Andrea, and Adriano Rampini, 2006, Capital reallocation and liquidity, *Journal of Monetary Economics*, 53, 369-399.
- Erickson, Timothy, and Toni Whited, 2000, Measurement Error and the Relationship between Investment and  $q$ , *Journal of Political Economy*, 108, 1027-1057.
- Erickson, Timothy, and Toni Whited, 2012, Treating Measurement Error in Tobin's  $q$ , *Review of Financial Studies*, 25, 1286-1329.

- Fresard, Laurent, and Philip Valta, 2013, Competitive Pressure and Corporate Investment: Evidence from Trade Liberalization, working paper, HEC Paris.
- Grossman, Sanford, and Oliver Hart, 1982, Corporate financial structure and managerial incentives, in John J. McCall, ed.: *The economics of information and uncertainty* (University of Chicago Press, Chicago).
- Guedes, Jose, and Tim Opler, 1996, The Determinants of The Maturity of Corporate Debt Issues, *Journal of Finance*, 51, 1809-1833.
- He, Zhiguo, and Konstantin Milbradt, 2013, Endogenous Liquidity and Defaultable Bonds, forthcoming, *Econometrica*.
- He, Zhiguo and Wei Xiong, 2012a, Rollover Risk and Credit Risk, *Journal of Finance*, 67, 391-430.
- He, Zhiguo and Wei Xiong, 2012b, Dynamic debt runs, *Review of Financial studies*, 25, 1799-1843.
- Hou, Kewei and David Robinson, 2006, Industry Concentration and Average Stock Returns, *Journal of Finance*, 61, 1927-1956.
- Hui, Chen, Yu Xu, and Jun Yang, 2013, Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads, MIT working paper, 2013.
- Johnson, Shane, 2003, Debt Maturity and the Effects of Growth Opportunities and Liquidity Risk on Leverage, *Review of Financial Studies*, 16, 209-236.
- Julio, Brandon, Woojin Kim, and Michael Weisbach, 2008, What Determines the Structure of Corporate Debt Issues?, NBER working paper.
- Kehoe, Timothy, and David Levine, 1993, Debt-constrained asset markets, *Review of Economic Studies*, 60, 865-888.
- Kehoe, Timothy, and David Levine, 2001, Liquidity constrained markets versus debt constrained markets, *Econometrica*, 69, 575-598.
- Kehoe, Timothy, and David Levine, 2008, Bankruptcy and collateral in debt constrained markets, in Roger E. A. Farmer, ed.: *Macroeconomics in the Small and the Large: Essays on Microfoundations, Macroeconomic Applications, and Economic History in Honor of Axel Leijonhufvud* (Edward Elgar).
- Kiyotaki, Nobuhiro, and John Moore, 1997, Credit cycles, *Journal of Political Economy* 105, 211-248.
- Leland, Hayne, 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance*, 49, 1213-1252.

- Leland, Hayne, 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance*, 53, 1213-1243.
- Leland, Hayne, and Klaus Bjerre Toft, 1996, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, *Journal of Finance*, 51, 987-1019.
- Mian, A., and J. Santos, 2011, Liquidity Risk, and Maturity Management over the Credit Cycle, working paper, Princeton.
- Milbradt, Konstantin, and Martin Oehmke, 2013, Maturity Rationing and Collective Short-Termism, working paper MIT and Columbia University.
- Midrigan, Virgiliu, and Daniel Xu, 2014, Finance and Misallocation: Evidence from Plant-level Data, *American Economic Review*, 104, 422-58.
- Moll, Benjamin, 2014, Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?, forthcoming, *American Economic Review*.
- Myers, Stewart C., 1977, Determinants of corporate borrowing, *Journal of Financial Economics*, 5, 147-175.
- Puterman, Martin, 2005, Markov decision processes: discrete stochastic dynamic programming. New Jersey: John Wiley and Sons.
- Rampini, Adriano, and S. Viswanathan, 2010, Collateral, risk management, and the distribution of debt capacity, *Journal of Finance*, 65, 2293-2322.
- Rampini, Adriano, and S. Viswanathan, 2013, Collateral and capital structure, *Journal of Financial Economics*, 109, 466-492.
- Scherr, Frederick, and Heather Hulburt, 2001, The debt maturity structure of small firms, *Financial Management*, 30, 85-111.
- Sequra, Anatoli and Javier Suarez, 2013, Recursive Maturity Transformation, working paper, CEMFI.
- Valta, Philip, 2012, Competition and the cost of debt, *Journal of Financial Economics*, 105, 661-682.
- Whited, Toni and Guojun Wu, 2006, Financial Constraints Risk, *Review of Financial Studies*, 19, 531-559.

# Biography

Wei Wei was born in Wuhan, the People's Republic of China on November 9, 1985. He graduated from the University of Mississippi with a Bachelor of Business Administration in Finance (summa cum laude) and a Bachelor of Sciences in Mathematics (summa cum laude) in 2008. He will get a Doctor of Philosophy in Business Administration (Finance) from Duke University in 2014 and start as an Assistant Professor of Finance at the University of Calgary in July 2014.